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Dr. A. C. D. CROMMELIN, PRESIDENT, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Cyril Jackson, Union Observatory, Johannesburg, S. Africa ;
J. E. Kerrich, M.Sc., University of the Witwatersrand, Johannesburg, S. Africa ;
Antonie Pannekoek, Professor of Astronomy, University of Amsterdam ;
Dhirendra Nath Saha, Extra-Assistant Superintendent, Survey of India, Karanpur, Dehra Dun, U.P., India ;
Sarupchand Mahanlal Shah, B.A., 7 Green Lane, London, N.W. 4 ;
and
Gustav Stracke, Ph.D., Recheninstitut, Berlin-Dahlem, Germany.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Mars F. Baumgardt, D.O.S., Curator of the Clark Observatory, 1208 Hollingsworth Building, 606 South Hill Street, Los Angeles, Cal., U.S.A. (proposed by F. C. Leonard) ;
Dr. Edwin F. Carpenter, Steward Observatory, University of Arizona, Tucson, Arizona, U.S.A. (proposed by A. E. Douglass) ;
Hendon Egerton Houghton, High Commissioner's Office, Cape Town, S. Africa (proposed by H. Horrocks) ;
Kenelm Jocelyn Townshend Hutchings, Lieut. R.N.R., Anafa, Degangwy, N. Wales (proposed by D. Larham) ;

Charles Vivian Jackson, Imperial College of Science and Technology, South Kensington, and 70 Addison Road, London, W. 14 (proposed by A. Fowler); and

Miss Violet Florence White, B.Sc., 3 Bonneville Gardens, Clapham Park, London, S.W. 4 (proposed by C. C. L. Gregory).

One hundred and forty-nine presents were announced as having been received since the last ordinary meeting, including, amongst others:—

Abbadia Observatory: Catalogue de 1343 étoiles, dont 720 fondamentales, comprises entre $+43^{\circ}$ et -26° , observées en 1919, 1920, 1921. Réduites à 1920.0.

Deuxième Catalogue comprenant 1332 étoiles, dont 711 fondamentales, de $+45^{\circ}$ à -26° , observées en 1922, 1923. Réduites à 1920.0.

Troisième Catalogue comprenant 1250 étoiles, dont 658 fondamentales, de $+45^{\circ}$ à -26° , observées en 1924, 1925. Réduites à 1925.0.

Observations, Tome 14.

R. T. Gould: Enigmas.

G. H. Hamilton: Mars at its Nearest.

Melbourne Observatory: Astrographic Catalogue, vol. 3, zones -69° , -70° .

The President announced the discovery, by Lowell Observatory, of a trans-Neptunian planet, and it was resolved to send the following telegram of congratulation to Lowell Observatory:—

“The President, Council, and Fellows of the Royal Astronomical Society in meeting assembled send the Lowell Observatory their heartiest congratulations on the great discovery of the trans-Neptunian planet.”

On the Calculation of the Relative Temperatures and Pressures existing at the Base of Sunspots. By R. M. Petrie.

(Communicated by Professor E. A. Milne.)

The purpose of this note is to present methods of calculating certain quantities connected with the structure of sunspots upon the assumption that known physical laws are applicable to the gaseous material composing the solar atmosphere. The discussion is divided into three parts, each one being a calculation of the parameters, pressure and temperature, with an additional degree of refinement in the approximations introduced each time.

The present investigation has as its origin a paper by H. N. Russell on "Cooling by Expansion in Sunspots" (*Ap. J.*, **54**, 293, 1921). This important paper is the first quantitative treatment of the equilibrium of the sunspot and the adjacent photosphere. For the sake of comparison and to make the method clear, Russell's computations are included with the other material here presented.

The mechanism of the spot formation is taken as the adiabatic expansion of a body of gas moving upward from some level in, or below, the solar photosphere. Due to the large area of the spot compared to its depth, the edge effects, such as conduction and radiation into and from the sides of the spot, are neglected. The following notation regarding the pressures and temperatures is adopted. At the zero level of the photosphere we use P_1, T_1 , to denote the pressure and temperature; the same quantities in the sunspot, at the same zero level, are represented by P_1' and T_1' . At the point called the base of the sunspot, from which the uprising material ascends, there is to be no difference between the spot and the surrounding material, and there we use P_0 and T_0 . The values of T_1 and T_1' are known from colorimetric measures. When T_0 and P_0 are found it is then possible to find the depth of the sun-spot from relations between temperature and depth below the solar surface which exist in the theory of radiative equilibrium. We now proceed to present the methods and calculations.

(1) This first method is that of Russell and is a recapitulation of his paper previously referred to. In this first treatment we use for the photosphere the relation $T \propto P^{\frac{1}{\gamma}}$ which follows from the theory of radiative equilibrium. For the sunspot, using the relations existing in the adiabatic expansion of a gas, we get $T \propto P^{\frac{\gamma-1}{\gamma}}$, where γ is, as usual, the ratio of the specific heats for the gaseous material. For the photosphere we then have

$$\frac{T_1}{T_0} = \left(\frac{P_1}{P_0}\right)^{\frac{1}{\gamma}} \quad . \quad . \quad . \quad . \quad (1)$$

and for the spot

$$\left(\frac{T_1'}{T_0}\right) = \left(\frac{P_1'}{P_0}\right)^{\frac{\gamma-1}{\gamma}} \quad . \quad . \quad . \quad . \quad (2)$$

The assumption is now made that $P_1 = P_1'$. This is an approximation and gives an approximate solution. In equation (2), $\left(\frac{P_1'}{P_0}\right)$ may therefore be replaced by $\frac{P_1}{P_0}$ and, combining (1) and (2), we have

$$\left(\frac{T_1'}{T_1}\right) = \left(\frac{P_1}{P_0}\right)^{\frac{3\gamma-4}{4\gamma}}$$

which, by means of equation (1), becomes

$$\left(\frac{T_1'}{T_1}\right) = \left(\frac{T_1}{T_0}\right)^{\frac{3\gamma-4}{\gamma}}$$

or

$$T_0 = T_1 \left(\frac{T_1}{T_1'} \right)^{\frac{\gamma}{3\gamma-4}} \quad (3)$$

Adopting specific values for T_1 and T_1' , the value of T_0 may then be found. For the gaseous mixture composing the solar atmosphere the value $\gamma = 5/3$, *i.e.* that for a perfect monatomic gas, seems to be most suitable. It is of interest, however, to compute the values of T_0 and P_0 resulting when the values of γ corresponding to more complex molecules are used. This was done for $\gamma = 3/2$ and for $\gamma = 7/5$, and hence some idea of the dependence of T_0 and P_0 on γ may be gained. Using colour temperatures for the photosphere and sunspot, we put $T_1 = 6000^\circ$ and $T_1' = 4000^\circ$ in equation (3), and so get the various values of T_0 corresponding to each value of γ used as shown in the table below. When we have T_0 in terms of T_1 and T_1' then it is possible to compute $\left(\frac{P_0}{P_1} \right)$ by means of formula (1) or formula (2), and the third column of the table shows the resulting values of this ratio. The values of the temperatures and pressures now follow :

γ .	T_0 .	P_0 .
5/3	12000°	16 P_1
3/2	20000°	130 P_1
7/5	100000°	80000 P_1

(2) In a series of lectures at Ann Arbor last summer, Professor E. A. Milne discussed the subject of convective cooling in sunspots and Russell's method, adding a treatment which takes into account the physical theory of the absorption coefficient. By permission of Professor Milne, his method is reproduced here and forms the second section. In this second method a slightly different law of radiative equilibrium is employed, making use of the optical depth τ . The same approximation regarding the pressures inside and outside the spot at the same level is made. The equation of radiative equilibrium here used is

$$T^4 = \frac{1}{2} T_e^4 (1 + \frac{3}{2} \tau) \quad (4)$$

where T_e is the effective temperature and T is the temperature at the optical depth τ . According to modern theory and experiment we may best write τ as

$$\tau = C_1 \int \frac{P dP}{T^{9/2}},$$

so that

$$\frac{d\tau}{dP} = \frac{C_1 P}{T^{9/2}} \quad (5)$$

From equation (4) we get

$$4T^3 \frac{dT}{dP} = \frac{3}{4} T_e^4 \frac{d\tau}{dP},$$

which, by (5), becomes

$$T^3 \frac{dT}{dP} = \frac{3}{16} \frac{T_e^4 C_1 P}{T^{9/2}} \quad \text{or} \quad T^{15/2} = C_2 P \frac{dT}{dP},$$

where

$$C_2 = \frac{3}{16} T_e^4 C_1.$$

By integration of this equation we obtain

$$\frac{2}{17} T^{17/2} = \frac{C_2}{2} P^2 + C_3.$$

The value of C_3 , the constant of integration, is determined by employing boundary conditions as follows: we let the boundary of the sun be that point where the pressure has fallen to zero and the temperature is T_s . We then get $C_3 = \frac{2}{17} T_s^{17/2}$, and may write our relations as

$$T^{17/2} - T_s^{17/2} = \frac{17}{4} C_2 P^2 \quad . \quad . \quad . \quad (6)$$

As before, the adiabatic expansion in the spot yields the relation

$$T \propto P^{\frac{\gamma-1}{\gamma}},$$

and we have the approximation $\frac{P_1'}{P_0} = \frac{P_1}{P_0}$ also holding. From equation (6) we may write

$$\frac{T_1^{17/2} - T_s^{17/2}}{T_0^{17/2} - T_s^{17/2}} = \left(\frac{P_1}{P_0} \right)^2 = \left(\frac{P_1'}{P_0} \right)^2,$$

and from the adiabatic law we have

$$\left(\frac{P_1'}{P_0} \right) = \left(\frac{T_1'}{T_0} \right)^{\frac{\gamma}{\gamma-1}},$$

so that the above equation becomes

$$\frac{1 - \left(\frac{T_s}{T_1} \right)^{17/2}}{\left(\frac{T_0}{T_1} \right)^{17/2} - \left(\frac{T_s}{T_1} \right)^{17/2}} = \left[\left(\frac{T_1'}{T_1} \right) \left(\frac{T_1}{T_0} \right) \right]^{\frac{2\gamma}{\gamma-1}} \quad . \quad . \quad . \quad (7)$$

Now from equation (4) we may express the temperature of any layer if we know the optical depth of the layer so that, for the boundary, where $\tau = 0$, we get

$$T_s^4 = \frac{1}{2} T_1^4,$$

and so $\left(\frac{T_s}{T_1} \right) = \left(\frac{1}{2} \right)^{1/4}$. Also, using the adopted values of photospheric

and spot temperatures we get $\left(\frac{T_1}{T_1'}\right) = 3/2$. Equation (7) may now be written

$$\frac{1 - \left(\frac{1}{2}\right)^{\frac{17}{8}}}{\left(\frac{2}{3}\right)^{\frac{2\gamma}{\gamma-1}}} = \left(\frac{T_1}{T_0}\right)^{\frac{2\gamma}{\gamma-1}} \left[\left(\frac{T_1}{T_0}\right)^{-\frac{17}{2}} - \left(\frac{1}{2}\right)^{\frac{17}{8}} \right]$$

and, replacing γ by $5/3$, $3/2$, and $7/5$ in turn, the following equations result:—

$$\left. \begin{aligned} \text{for } \gamma = 5/3, & \quad \left(\frac{T_1}{T_0}\right)^{\frac{17}{2}} + 25.530 \left(\frac{T_1}{T_0}\right)^{\frac{5}{2}} - 4.3620 = 0 \\ \text{,, } \gamma = 3/2, & \quad \left(\frac{T_1}{T_0}\right)^{\frac{17}{2}} + 38.294 \left(\frac{T_1}{T_0}\right)^{\frac{3}{2}} - 4.3620 = 0 \\ \text{,, } \gamma = 7/5, & \quad \left(\frac{T_1}{T_0}\right)^{\frac{17}{2}} + 57.442 \left(\frac{T_1}{T_0}\right)^{\frac{3}{2}} - 4.3620 = 0 \end{aligned} \right\} \quad (7a)$$

After solving equations (7a) for $\left(\frac{T_1}{T_0}\right)$, we may then proceed to evaluate P_0 in terms of P_1 from the relations $\left(\frac{P_0}{P_1}\right) = \left(\frac{T_0}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$. The results are collected in the table which immediately follows:

γ .	$\frac{T_1}{T_0}$.	T_0 .	P_0 .
5/3	0.6031	9900°	3.5 P_1
3/2	0.4194	14300°	13.6 P_1
7/5	0.1793	33500°	409.7 P_1

It will be noticed that the change in the values of the pressures and temperatures with γ is not so marked here as in the first treatment.

(3) This third method arose from discussions of the matter with Professor Milne, during which I suggested modifying the treatment so as to take into account the pressure-differences existing between the top of the photospheric column and the top of the sunspot column. Since the spot is cooler than the surrounding photosphere, the pressure of the gaseous material at its surface will be less than that of the photosphere material. The base of the sunspot column will, however, still be at the same temperature and pressure as the photospheric gases at that level. The following calculations represent an attempt to evaluate this pressure-difference between spot and photosphere at the top. The equation of radiative equilibrium utilized in the treatment given in section (2) is also employed here. We had previously

$$T^{\frac{17}{2}} = \frac{17}{4} C_2 P^2 + C_3$$

as the relation between the pressure and the temperature in the atmosphere under radiative equilibrium. Let us choose our limits here so

that $P = 0$ when $T = 0$, so that for the general atmosphere we have

$$T \propto P^{\frac{1}{\gamma}} \quad (8)$$

while for the spot the usual relation

$$T \propto P^{\frac{\gamma-1}{\gamma}}$$

holds. The same results are very closely obtained if we do not choose our zero point so that C_3 is neglected. Professor Milne has shown (*Phil. Trans.*, 228, 456, 1929) that as soon as any appreciable depth in the photosphere is reached then we have

$$T \propto P^{\frac{1}{\gamma}},$$

so that the neglect of C_3 is justified. From formula (8) we may write

$$\left(\frac{T}{T_0}\right) = \left(\frac{P}{P_0}\right)^{\frac{1}{\gamma}} \quad (8a)$$

applying to the photospheric gases, while from the adiabatic law the form for use in the sunspots is

$$\left(\frac{T'}{T_0}\right) = \left(\frac{P'}{P_0}\right)^{\frac{\gamma-1}{\gamma}} \quad (8b)$$

and so we get

$$\frac{T'}{T} = P_0^{\frac{17-13\gamma}{17\gamma}} \left(\frac{(P')^{\frac{\gamma-1}{\gamma}}}{P^{\frac{1}{\gamma}}} \right) \quad (8c)$$

In any atmosphere, where the density is ρ , the force of gravity g , and the height h , we may write, neglecting radiation-pressure,

$$dP = -g\rho dh$$

as the equation giving the variation of pressure with depth in the solar atmosphere. Hence we may write

$$\frac{dP}{dP'} = \frac{\rho}{\rho'} = \frac{PT'}{P'T} \quad (9)$$

where spot and the surrounding atmosphere are both considered. By means of formula (8c) equation (9) may be written

$$\frac{dP}{dP'} = \frac{(P)^{\frac{1}{\gamma}} (P_0)^{\frac{17-13\gamma}{17\gamma}}}{(P')^{\frac{1}{\gamma}}},$$

which, upon integration, gives

$$\frac{17}{4}(P)^{\frac{1}{\gamma}} = \frac{\gamma}{\gamma-1}(P_0)^{\frac{17-13\gamma}{17\gamma}} (P')^{\frac{\gamma-1}{\gamma}} + C_4 \quad (10)$$

and, using our definite values, first at our zero level and then at the level of the sunspot base, we may write the equation (10) as

$$\frac{17(\gamma - 1)}{4\gamma} \left[1 - \left(\frac{P_1}{P_0} \right)^{\frac{1}{\gamma}} \right] = \left[1 - \left(\frac{P_1}{P_0} \right)^{\frac{\gamma-1}{\gamma}} \right]. \quad (10a)$$

This equation may be further transformed by means of (8a) and (8b) to yield

$$\frac{17(\gamma - 1)}{4\gamma} \left[1 - \frac{T_1}{T_0} \right] = \left[1 - \frac{T_1}{T_0} \right] \quad (11)$$

Putting in our several values of γ , as before, we get three equations which will yield values of T_0 in terms of the adopted values of $T_1 = 6000^\circ$ and $T_1' = 4000^\circ$. These equations are:

$$\begin{aligned} \text{for } \gamma = 5/3, \quad T_0 &= \frac{17}{4} T_1 - \frac{10}{7} T_1', \\ \text{,, } \gamma = 3/2, \quad T_0 &= \frac{17}{5} T_1 - \frac{12}{5} T_1', \\ \text{,, } \gamma = 7/5, \quad T_0 &= \frac{17}{3} T_1 - \frac{14}{3} T_1'. \end{aligned}$$

Having obtained T_0 in terms of T_1 and T_1' , then from (8a) and (8b) the values of $\frac{P_1}{P_0}$ and $\frac{P_1'}{P_0}$ may be computed. The results are shown in the table below:

γ .	T_0 .	P_0 .	P_1' .
5/3	8800°	5.2 P_1	0.72 P_1
3/2	10800°	12.2 P_1	0.62 P_1
7/5	15300°	53.4 P_1	0.50 P_1

It may be seen from this discussion that the changes in T_0 and P_0 with γ are still less marked than in the second method. It is of interest to note in particular that for the case of the value $\gamma = 5/3$ we get $P_1' = 0.72 P_1$, making the pressure in the sunspot at its surface about 7×10^{-5} atmospheres. We may also compute the velocity of rotation to which this pressure-difference would correspond for a spot on the solar equator. The equation to be used is $\frac{\partial P}{\partial r} = \frac{\rho v^2}{r}$, v being the velocity of rotation of the gases around the vortex. Considering the total change of pressure between spot and photosphere we have, approximately,

$$P_1 - P_1' = \rho v^2 \quad \text{or} \quad v = \sqrt{\frac{(1 - 0.72)RT_1}{\mu}}$$

by use of the gas laws, μ being the mean molecular weight of the material. Using $T_1' = 4000^\circ$, $\mu = 20$, this equation yields $v = 0.8$ km. sec.⁻¹. This is the maximum velocity for a spot on the equator, and does not consider any radial outflow from the spot but merely deals with a rotating column.

As previously suggested, it is now possible to express the depth of sunspots from data giving the temperature in the photosphere at a given

level. Professor Milne has given such a table (*Phil. Trans.*, **228**, 458, 1929) from which the depth in the photosphere corresponding to our T_0 may be found. These depths, below the level where $T_1 = 6000^\circ$, were found to be

γ .	Method I.	Method II.	Method III.
5/3	49 kms.	37 kms.	27 kms.
3/2	102 "	65 "	41 "
7/5	..	191 "	70 "

a temperature gradient of 151° per km. being assumed for the high temperatures.

From these studies a few conclusions may be drawn, although it should be borne in mind that the methods and results are approximate ones and that the values obtained are not to be considered as precise but rather indicative of the order of the quantities. It seems, however, that the pressure and temperature at the base of the spots become less sensitive to changes in γ as approximations of increasing order of refinement are employed. Further, the base pressures and temperatures increase with a decrease in γ , so that, for departures from monatomic gas conditions, the sunspots would have to originate at greater depths below the photosphere in order that the observed temperature-differences between the spot and the photosphere be maintained.

In closing, I should like to express my thanks to Professor Milne for his kindly interest and advice during the progress of these calculations. To Professor W. C. Rufus my thanks are also due for advice and helpful criticism.

Ann Arbor, Michigan :
1930 February.

Preliminary Note on the Structure of Sunspots. By E. A. Milne.

1. Since my discussions of last summer with Mr. Petrie, described in his foregoing note, I have been independently examining the question of sunspot structure. Mr. Petrie's note affords me an opportunity of making a preliminary statement of my results. The results confirm those of Mr. Petrie as regards pressures, temperatures, and linear depth of the spot, but they provide further a theoretical evaluation of the velocity of outflow first observed by Mr. Evershed.*

2. If we take a broad survey of the phenomena characteristic of sunspots we may classify them thus: (1) Darkness of umbra, (2) continued existence over considerable periods, (3) periodicity, (4) latitude of outbreak, (5) frequent occurrence in pairs, (6) radial motion, (7) magnetic fields. By way of comment on these it may be mentioned (1) that spectroscopic investigations attribute to the umbra a temperature of 4000° , which is confirmed by comparison with stellar spectra. A temperature of 3900° has recently been found by Ambarzumian and

* *M.N.*, **69**, 454, 1909; **70**, 217, 1910.

Kosirev,* from analysis of the continuous spectrum, and a number of early observations of the intrinsic brightness of the umbra relative to the photosphere agree with this as regards order of magnitude.† Absolute measures of the effective temperature of the umbra are much to be desired—we do not in fact know whether the umbral temperature is or is not another “solar constant.” (2) Spots persist, sometimes for months. They are therefore structures possessing a measure of stability. (3) There are three planes of reference in the sun—(a) the ecliptic, (b) the equator of rotation, (c) the magnetic equator. The correlation of outbreak with latitude, measured with regard to the rotational equator, clearly indicates solar rotation as the origin of spots. (4) Evershed has found that certain gases are moving radially outward from the umbra with a mean speed of 1.8 kms./sec. St. John, in confirming this,‡ found that at low levels the gases are moving outwards, at high levels inwards. (5) The magnetic fields are of the order of 1300–3000 gauss.§

A complete theory of sunspots should be able not only to account qualitatively for the facts, but also, given only certain photospheric constants such as the photospheric values of temperature and gravity and the mean molecular weight, to predict that spots should be some 2000° cooler than the photosphere, that the gases at low levels are moving outwards at about 2 kms./sec., and that spots are the seat of magnetic fields of the order of 2000 gauss. Why is a spot 2000° cooler than the photosphere and not 500° or 4000° cooler? Why should the velocity of outflow be 2 kms./sec. and not 0.2 or 20 kms./sec.? And why should the field strength be 2000 gauss and not 200 gauss or 20,000 gauss?

Theories of sunspots so far put forward may be divided into (a) frontal attacks, (b) attacks with a limited objective. Of the former, the earlier examples are fully described in Bosler's *Theories Modernes du Soleil*; the most notable recent example is the sea-serpent hypothesis of V. Bjerknes.|| All theories of this type either meet difficulties or involve *ad hoc* hypotheses. It appears to me, in view of the past history of such theories, that no frontal attack is likely to succeed at present. The right theory, when the time is ripe, will explain the diverse phenomena without effort. No such theory is at present forthcoming—periodicity properties, for example, are far from an explanation. In the absence of such a theory we may ask for theories with a limited objective. What limited objectives are feasible?

The most feasible limited objective would appear to be to account for the persistence of spots. Accepting that there is some unknown cause which brings sunspots into existence, we may ask for an explanation of how they persist for considerable periods, how they have a quasi-

* *A.N.*, 233, 107, 1928. I owe this reference to Professor Stratton.

† W. E. Wilson, *M.N.*, 55, 457, 1895; Evershed, *Ap. J.*, 5, 249, 1897; Schwarzschild and Villager, *ibid.*, 23, 345, 1906. I owe these references to Professor Turner and Mr. Evershed.

‡ *Ap. J.*, 37, 322, 1913; 38, 341, 1913.

§ Hale, *Ap. J.*, 28, 100 and 315, 1908, etc.

|| *Ap. J.*, 64, 93, 1926.

permanent existence. Such a theory, dealing not with spot-origin but with spot-structure, ought to be able to account for the observed temperature, the observed radial velocity, and the observed field-strength.

The researches of Evershed and St. John give us a consistent picture of the upper part of a spot. Gases are flowing out at a certain level. The deficiency of radiation-pressure above a spot, combined with the reduction in gas-pressure* in the umbra, cause a falling-in or sucking-in of the chromospheric gases in the manner first described by S. R. Pike †; they flow inwards at high levels, then downwards towards the umbra, then presumably join the outward flow. St. John has shown ‡ by calculation that the mass thus inflowing is inadequate to account for the total horizontal outflow. Hence we are bound to assume a rise of

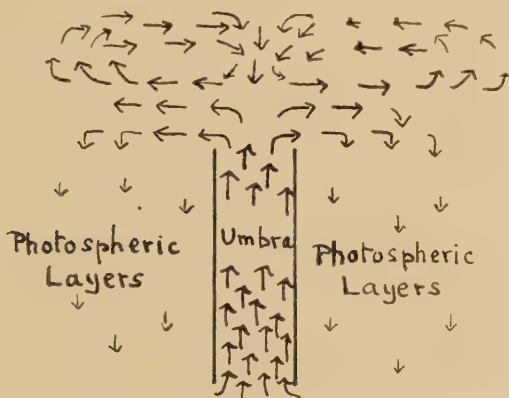


FIG. 1.—The flow in a sunspot.

gases in the umbra from below, through layers already in radiative equilibrium. In this uprush they will expand, and therefore suffer some adiabatic cooling relative to the surrounding gases. Hence the temperature in the umbra will be below that in the adjacent photosphere. We thus have a picture of a spot of the kind first described by St. John, and illustrated in fig. 1. But this picture does not of itself account for the observed difference of temperature between umbra and photosphere, for the ascending gases might have come from any depth, and so the temperature-difference might have any value. This is also the model of a sunspot taken over by Bjerknæs,§ with vorticity added. But Bjerknæs does not show why the temperature-difference and the radial velocity should have their observed values.

There are, however, two important contributions to the quantitative

* See Mr. Petrie's paper, and also *infra*.

† *M.N.*, **87**, 56, 1926.

‡ *Ap. J.*, **37**, 343, 1913.

§ I distinguish between Bjerknæs' model of a spot, and his general theory of internal circulation.

study of the spot on this model, namely those of Russell* and Rosseland.† Russell took account of the adiabatic cooling (neglecting the heating effects of radiation), and used the observed temperature-difference to estimate the pressure and temperature at the base of the rising column. Rosseland calculated the change in net flux of radiation between bottom and top of the umbral column, and estimated the vertical thickness of a spot as not less than 50 kms. The methods used by Russell and Rosseland are capable of considerable extension.

We may idealise the circulation in the lower part of a spot into the rectangular circuit ABCD of fig. 2, which shows a vertical cross-section of a spot (lower part). Gases ascend from A to B, expanding and cooling. The cooling is less rapid than adiabatic, since there is heating by radiation from below, besides (as Russell pointed out) heat evolved by chemical and ionic recombination. The gases then travel hori-

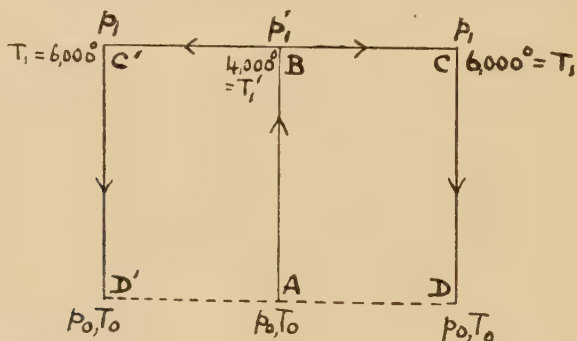


FIG. 2.—Vertical cross-section of spot-circulation (diagrammatic).

zontally from B to C, increasing in temperature from $T_1' = 4000^\circ$ to $T_1 = 6000^\circ$ (BC is a typical radius of the spot when seen from above). They then slowly descend from C to D. The path from D to A we need not at present inquire into. AB is actually the axis of a rising core of gas, CD is a typical generator of the various cylinders of descent (annular regions co-axial with AB). The area of ascent corresponding to AB is very small compared with the annular area of descent corresponding to CD and C'D'. Hence the velocity of descent in CD and C'D' is negligible compared with the velocity of ascent in AB. We may in fact treat the column CD as in hydrostatic equilibrium and its temperature distribution as radiative.

The distribution of velocity, temperature, pressure, and density in the column AB is a perfectly definite hydrodynamical problem, of which the equations may be readily written down. A derivation of the equations is given in Rosseland's paper. I have given an alternative derivation of the equations, in a paper communicated to the

* *Ap. J.*, **54**, 293, 1921.

† *Ibid.*, **63**, 356, 1926.

Quarterly Journal of Mathematics (Oxford Series) of which No. 1 will appear shortly; further mathematical analysis of the sunspot problem will be given on a later occasion. Here I wish to describe a treatment on more elementary physical lines.

The problem is, can we find a state of motion ABCD which is steady in the hydrodynamical sense? If so, we shall have accounted for the semi-permanence of sunspots. We know one permanent distribution capable of existing on the sun, namely, combined mechanical and radiative equilibrium. This must hold over the greater part of the sun's surface. But we wish to examine whether locally, in isolated regions, we can have the circulation ABCD. No energy is lost in this circulation, apart from effects of viscosity which we can neglect on a first approximation. The potential energy gained in the ascent AB is given up in the descent CD. The motions of the fluid particles in the circuit are formally reversible, but it is difficult to see how the system could ever start in the reverse direction, for this would mean that the low-temperature umbral region would have to act as cause and not effect. The possibility of a permanent state arises from the fact that the rate of increase of temperature down BA will exceed that down CD, so that there will be a level at which the temperature in the former column (allowing for its starting handicap) will overtake that in the latter. It is clear that the kinetic energy of the gas emerging at B will be reduced practically to zero at C, being converted into thermal energy of compression. We see, in fact, that there must be a horizontal pressure gradient along BC, B being a region of low pressure, C of high pressure. The gases emerging at B are brought to rest in moving against the pressure gradient directed along CB. The change of temperature,* 4000° to 6000°, along BC will be the adiabatic increase consequent on adiabatic compression, of gas with velocity v , from the pressure p_1' at B to the state of approximate rest at pressure p_1 at C, increased by the heating by radiation in passing from B to C over a layer beneath having a horizontal temperature gradient. The necessary formula is the Bernoullian integral of hydrodynamics, modified by a radiation term. In the absence of this term the formula runs, in a standard notation,

$$\frac{1}{2}v^2 + \frac{p_1'}{\rho_1} + C_v T_1' = \frac{p_1}{\rho_1} + C_v T_1. \quad (1)$$

where v is the velocity at B. If γ is the ratio of specific heats, this becomes

$$\frac{1}{2}v^2 + \frac{\gamma}{\gamma - 1} \frac{R}{\mu} T_1' = \frac{\gamma}{\gamma - 1} \frac{R}{\mu} T_1. \quad (2)$$

where μ is the molecular weight.

The base A will be at a much higher pressure than the top B, but

* I omit here consideration of the interesting speculation whether we should not rather take the temperature at C to be the temperature of the faculae. Possibly faculae are hotter than the photospheric layers, simply in virtue of the adiabatic compression undergone by the gases along BC.

smaller than D, for it is a region of velocity. We have the condition that the kinetic energy generated at A, modified by the changes in kinetic energy undergone along AB, must be such as to give a kinetic energy at B which will just suffice to balance the pressure gradient along CB. The changes in kinetic energy along AB, allowing for heating by radiation, can be calculated by a generalisation of the Bernoullian integral which I have given in the paper in *Q.J.M.(O.)* above-mentioned. Lastly the kinetic energy generated at A must be connected with the pressure difference between D and A.

The unknowns are the velocities at A and B, the linear depth AB, and the pressures and temperatures at A, B, and D. The data are the photospheric constants, namely, the temperature and pressure at C (the two being interconnected by the theory of the photosphere) together with the surface gravity and mean molecular weight. If a unique circulation is possible as a stable configuration, the various relations existing should suffice to deduce the unknowns from the data, and so predict the umbral temperature and the Evershed radial velocity of outflow.

In order to see whether we have enough relations to determine the unknowns, it is advantageous to idealise the problem one stage further as a first approximation. If the rate of ascent along AB is sufficiently rapid we may neglect the heating by radiation and consider adiabatic cooling, following Russell. On the other hand, if the rate of ascent is sufficiently slow, we may ignore the effect of change of kinetic energy on the pressure-gradient and determine the pressure-gradient along AB from hydrostatic considerations. It is a legitimate assumption to suppose as a first approximation that both conditions are fulfilled. This assumption will at least give us insight into what we may expect from a complete scheme of exact equations. In any case the pressure difference between A and B will be principally governed (as is that between D and C) by the hydrostatic head due to the weight of the column, so that changes of kinetic energy will not be of substantial significance in deducing the pressure difference. Lastly, we will neglect the pressure difference between A and D, thus using an equation $p_A = p_D = p_0$ instead of the Bernoullian equation connecting p_A and p_D . In this idealisation of the problem we have precisely as many equations as in the actual problem; we have cut out certain unknowns and an equal number of equations.

Our picture now is: a column AB in adiabatic equilibrium balances a column DC in radiative equilibrium. Since the column AB is the colder, and the pressure at A equals the pressure at D, therefore the pressure at B will be less than the pressure at C*; for the column AB will have a lower centre of mass than the column DC. Suppose, now, we begin with a definite pressure p_1 at C and temperature $T_1 = 6000^\circ$, descend the column CD an arbitrary distance x , determine the pressure and temperature (p_0, T_0) at x (from the equilibrium of the radiative column CD), and ascend from this pressure and temperature

* This circumstance, the origin of the investigation, was first pointed out to me by Mr. Petrie in discussions at Ann Arbor.

an equal distance x along the adiabatic column AB. We shall arrive at a certain pressure and temperature, p_1' and T_1' , at B. By choosing x properly we can arrange so that $T_1' = 4000^\circ$. The pressure p_1' is thus determined. We have used the observed temperature difference "*photosphere — spot*" to compute the pressure-difference "*photosphere — spot*." Such a calculation has been carried out by Mr. Petrie in the preceding note. As a by-product we determine the depth BA of the umbral column and the pressure and temperature at its base.

Consider now the conditions connecting B and C. If the motion from B to C were adiabatic, we could calculate the velocity at B which would just be reduced to rest in passing adiabatically from the temperature at B to that at C, by formula (2) above. We could thus use the observed $\Delta T = T_1 - T_1'$ to compute the Evershed effect. We can also compute the temperature-difference $\Delta'T$ which corresponds adiabatically to the compression from p_1' to p_1 . This will not in general equal the required temperature-difference $\Delta T = 2000^\circ$. $\Delta'T$ will be less than ΔT , since the gases, in addition to their adiabatic heating, will be heated from below. We can now use the discrepancy between the adiabatically calculated $\Delta'T$ and the observed ΔT to determine the departure from adiabatic compression along BC—to determine in fact the external heating—and so revise our velocity calculation. The necessary formula is somewhat complicated to deduce, and depends on the general theory of the paper in *Q.J.M.(O.)*, and so I suppress it here, reserving it for a later communication.

Lastly, the external heating thus computed from the observed ΔT must be equal to the actual heating that would be produced in the gas flowing from B to C by the stream of radiation leaving the layers below, which possess a horizontal temperature-gradient. This last equality will not automatically be satisfied. It will only be satisfied for some particular temperature T_1' , given T_1 . If T_1' is too low, the heating will be insufficient; if T_1' is too high, the heating will be too great. If we could determine the temperature T_1' which just satisfies the conditions we should have evaluated from purely photospheric data the temperature-difference "*photosphere — spot*." Thus our circuital equalities provide just sufficient relations to fix all the unknowns, and thus predict the umbral temperature and the Evershed velocity. We should then revise our calculations to take account of the effects neglected, namely, the heating of the rising umbral column by radiation and the pressure effects due to kinetic energy. I have not yet carried out the calculations sketched in this paragraph, but the difficulties seem only technical. I now communicate my results for the earlier calculation, which amount to a determination of the depth of the umbral column, the pressure-difference "*photosphere — spot*," and the Evershed velocity from the observed ΔT .

Mr. Petrie, following Russell, has used the classical adiabatic relation for the column AB. But the presence of radiation makes a material modification. Adiabatic equilibrium* in the presence of radiation-

* Adiabatic motion in the presence of radiation-pressure involves similar features.

pressure is a curious phenomenon which I have discussed in detail in the paper already referred to in *Q.J.M.(O.)*. The main results are that down a column in adiabatic equilibrium the density increases asymptotically towards a maximum, radiation-pressure becomes increasingly dominant to gas-pressure and p/ρ^2 increases. Taking account of these effects, I find that the temperature no longer increases linearly with depth—the gradient decreases. Fig. 3 shows the calculated tempera-

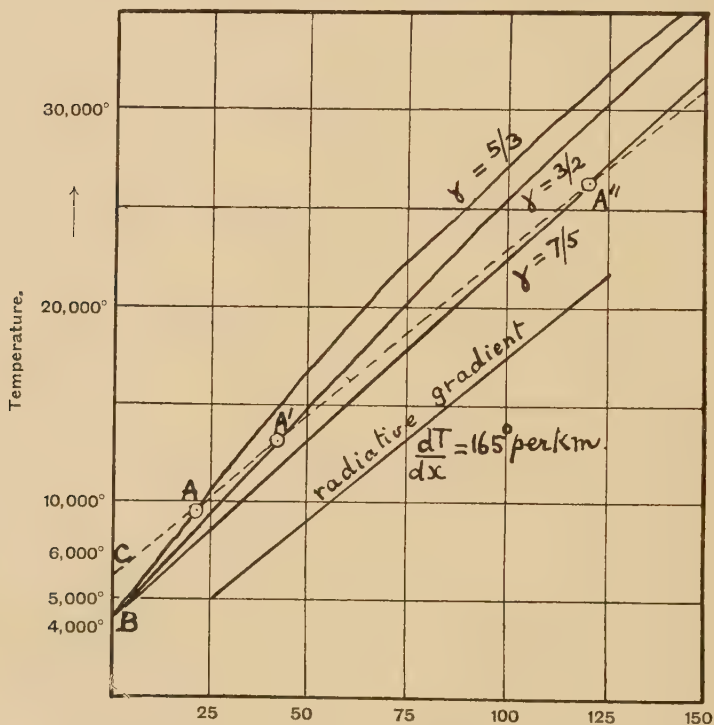


FIG. 3.—Determination of base of umbra.

ture distribution for $p_1' = 50$ dynes cm^{-2} , $T_1' = 4000^\circ$, $\mu = 20$, for three different values of γ , namely, $\gamma = 5/3$, $\gamma = 3/2$, $\gamma = 7/5$.

On the other hand, in radiative equilibrium the photospheric layers are the seat of a temperature distribution which is approximately linear. The gradient is readily calculated.* Through the point C ($x = 0$, $T = T_1 = 6000^\circ$) draw a straight line at the appropriate gradient (165° per km.). It intersects the adiabatic curves in points A, A', A'' (the lettering in fig. 3 corresponds to that in fig. 2). These might equally well have been labelled D, D', D''—they correspond to the base of the umbral column or the equivalent level in the radiative column. We can read off the temperature and depth at A from the

* See, e.g., *Phil. Trans.*, 228 A, 457, 1929.

graph. Along the radiative straight line CA, p/p_R is approximately constant (p_R = radiation pressure); along the adiabatic curves BA, etc., p/p_R is variable, but it is known from the calculations which have gone to the making of these curves. Hence p/p_R is known at A (or A' or A'') and hence at C. But p_R at C may be calculated from $T_1 = 6000^\circ$. Hence p is known at C. This evaluates p_1 corresponding to the assumed value $p_1' = 50$ dynes cm.⁻². (By trial and error the results could have been found for an assumed value of p_1 , i.e. we could have computed p_1' for an assumed p_1 .)

The results are given in the following table. It will be seen that they are in general agreement with Mr. Petrie's results. In particular, the values of p_1'/p_1 are closely the same as Mr. Petrie's.

γ .	T_0 .	p_0 (dynes cm. ⁻²).	p_1 (dynes cm. ⁻²).	p_1' (dynes cm. ⁻²).	p_1'/p_1 .	Depth (km.).
$\frac{5}{3}$	9,800°	460	64.3	50	0.78	21
$\frac{3}{2}$	13,000°	1,620	73.3	50	0.68	42
$\frac{7}{6}$	26,200°	32,400	89.1	50	0.56	120

Mr. Petrie, acting on my suggestion, has used his pressure-differences to evaluate the rotational velocity which the column could possess, assuming a cyclonic structure. Later hydrodynamic calculations of my own show that the outflow effect is dominant over the rotation—the cyclonic motion set up by the solar rotation is negligible compared with the velocity of outflow. This is in agreement with the bulk of the observational material, though it does not exclude the possibility of spots possessing occasionally a cyclonic structure. The point is that outflow is a necessary part of spot-structure, cyclonic rotation an accidental one. The Bernoullian energy integral is easily adapted to motion relative to a rotating frame, and is then still substantially equation (2) above. For adiabatic outflow from $T_1' = 4000^\circ$ to $T_1 = 6000^\circ$ the calculated velocities are as follows (μ has been taken to be 20):—

γ .	v . (kms. sec. ⁻¹).
$\frac{5}{3}$	2.0
$\frac{3}{2}$	2.2
$\frac{7}{6}$	2.4

The temperature-differences ΔT computed for adiabatic flow from p_1' to p_1 , using the above values of p_1' and p_1 , are approximately:

γ .	ΔT .
$\frac{5}{3}$	500°
$\frac{3}{2}$	600°
$\frac{7}{6}$	750°

These are less than the observed $\Delta T = 6000^\circ - 4000^\circ = 2000^\circ$, as was expected. The difference has to be made up by heating from below. Utilising the discrepancy to estimate the heating from below

and so revising our velocity calculation, we find for the revised velocities of outflow :

γ .	v . (kms. sec. ⁻¹).
$\frac{5}{3}$	1.0
$\frac{5}{2}$	1.3
$\frac{7}{5}$	1.6

The mean velocity of outflow observed by Evershed was 1.8 kms. sec.⁻¹. It will be seen that we have hit off the observed order of magnitude. Here an important point is to be noticed. Both in the calculation of the linear depth of the base of the umbra and in the calculation of outflow velocities it is necessary to assume a value for the mean molecular weight. I have taken $\mu = 20$ as representative. If, following Russell,* we assume the gas to be largely hydrogen, so that μ is reduced to, say, 2, the velocities calculated will be increased in the ratio $(20/2)^{\frac{1}{2}}$, i.e. by a factor of something over 3. This would give velocities much larger than the observed velocity. I do not press this point, but it is clear that ultimately analysis of outflow velocities will give exact information as to the mean molecular weight of the gases concerned.

With reference to the calculated depths of the umbral column, these may appear to be very small. But they are in agreement with estimates by Bjerknes, Rosseland, and Ambarzumian and Kosirev. It would therefore seem that the umbral region has the shape of a penny rather than that of a column; the depth is very small compared with the area.

Is there any chance of explaining magnetic effects with this model of a sunspot? Since Rosseland's paper of 1925† the significant new discovery in electro-magnetism has been the attribution of a spin and a magnetic moment to the electron. This fully accounts for the structure of multiplets in spectra, and it has been shown by Dirac that these properties can be naturally accounted for by relativity. May not these same properties, when properly woven into the solar situation, perhaps account for the magnetic field of sunspots and of the sun itself?

Oxford : 1930 *March* 3.

The Relative Variations in the Extents of Bases and in the Heights of Prominences accompanying Variations in their Areas. By P. R. Chidambara Iyer, B.A.

The profile areas of prominences on the sun's limb, their heights above the chromosphere, and the extents of their bases on the chromosphere are being systematically measured at Kodaikanal, and data of the mean daily areas, mean heights, and mean extents are available for each half-

* *Ap. J.*, **70**, 11, 1929.

† *Ap. J.*, **62**, 387, 1925.

year from 1905 to 1928. These data are published regularly in the half-yearly bulletins of prominence summaries issued from Kodaikanal. From the year 1923, under the auspices of the International Astronomical Union, Kodaikanal has been incorporating statistics derived from photographs of foreign observatories also, and, accordingly, the abstract for each half-year published in Table I. in the prominence bulletins from that year onwards contains the means derived from the data of all the co-operating observatories. For the sake of uniformity, however, the figures used in this investigation are those from the Kodaikanal records only.

The method of derivation of the data discussed here may be briefly summarised. The daily prominence photographs are examined for their quality and corrections are applied to incomplete or imperfect records, such that the total number of days in a half-year on which photographs were secured get counted as a smaller number of effective days. The mean daily areas are derived by dividing the total profile areas in each half-year by the appropriate number of effective days, while the sum of the greatest heights reached by individual prominences and the sum of the lengths of their bases divided by the total number of prominences observed give the mean height and the mean extent respectively.

It occurred to me that a parallel study of the changes in the extents of the bases and in the heights, accompanying changes in areas, might yield fruitful results, by revealing the nature and magnitude of some of the forces that come into play in the fluctuations of solar activity as reflected in the prominences. For instance, if, as the solar cycle advances, we get taller and taller prominences, we can legitimately infer that there is an increasing force or thrust acting outwards from the sun. The bases of prominences increasing in extent might, on the other hand, indicate that an increasing force is acting tangentially to the surface of the sun, or in other words, that the phenomenon of an increase in solar activity is essentially one of spread, its magnitude in the direction of longitudes being revealed by the increased extent in the prominence bases.

The bases are usually expressed in degrees on the circumference of the sun, whereas the heights are measured by the angles which they subtend at the earth. In order to make the bases comparable with the heights, the former were converted into equivalent apparent angles, on the basis of the sun's apparent semidiameter being taken as 16 minutes of arc. The following table contains the mean daily areas in square minutes and the mean bases and mean heights in minutes of arc from 1905 to 1928.

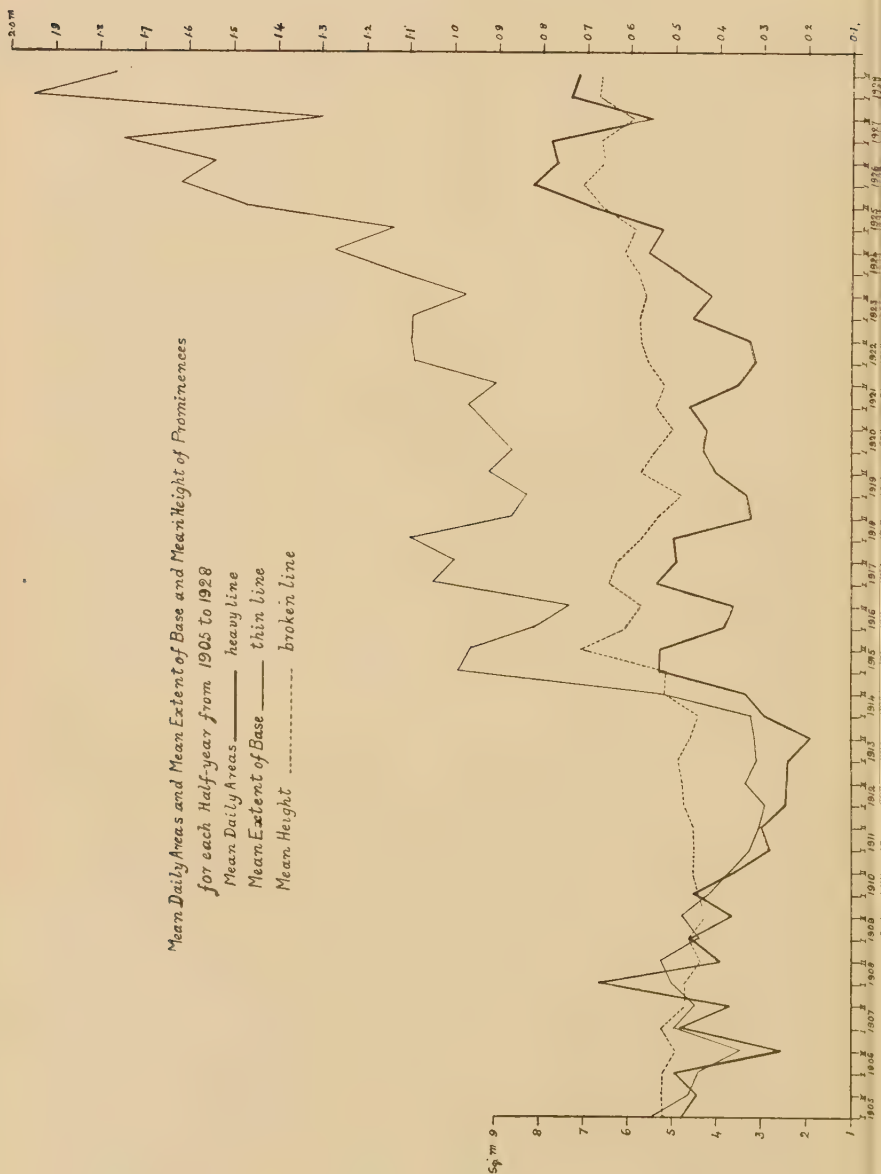
Years.	Mean Daily Areas of Promi- nences, sq. min.	Percentage Departure over Previous Half-Year.	Mean Extent of Base, min.	Percentage Departure over Previous Half-Year.	Mean Height, min.	Percentage Departure over Previous Half-Year.
1905 I. half	4.80		.550		.522	
II. "	4.46	- 7.1	.466	- 15.3	.523	+ 0.2
1906 I. "	4.94	+ 10.8	.444	- 4.7	.522	- 0.2
II. "	2.59	- 47.6	.349	- 21.4	.497	- 4.8
1907 I. "	4.81	+ 85.7	.497	+ 42.4	.525	+ 5.6
II. "	3.71	- 22.9	.452	- 9.1	.472	- 10.1
1908 I. "	6.67	+ 79.8	.502	+ 11.1	.473	+ 0.2
II. "	3.93	- 41.1	.527	+ 5.0	.437	- 7.6
1909 I. "	4.60	+ 17.0	.441	- 16.3	.462	+ 5.7
II. "	3.68	- 20.0	.480	+ 8.8	.428	- 7.4
1910 I. "	4.51	+ 22.6	.421	- 12.3	.442	+ 3.2
II. "	3.61	- 20.0	.371	- 11.9	.452	+ 2.3
1911 I. "	2.83	- 20.5	.326	- 12.1	.450	- 0.4
II. "	3.00	+ 6.0	.304	- 6.7	.452	+ 0.4
1912 I. "	2.48	- 17.3	.293	- 3.6	.473	+ 4.6
II. "	2.43	- 2.0	.335	+ 14.3	.477	+ 0.8
1913 I. "	2.42	- 0.4	.310	- 7.5	.487	+ 2.1
II. "	1.92	- 20.7	.315	+ 1.6	.460	- 5.5
1914 I. "	2.93	+ 52.6	.321	+ 1.9	.442	- 3.9
II. "	3.34	+ 14.0	.522	+ 62.6	.518	+ 17.2
1915 I. "	5.27	+ 57.8	.985	+ 88.7	.517	- 0.2
II. "	5.29	+ 0.4	.968	- 1.7	.708	+ 36.9
1916 I. "	3.88	- 26.7	.815	- 15.8	.608	- 14.1
II. "	3.65	- 5.9	.739	- 9.3	.572	- 5.9
1917 I. "	5.36	+ 46.8	1.041	+ 41.1	.643	+ 12.4
II. "	4.95	- 7.6	.999	- 4.0	.625	- 2.8
1918 I. "	5.00	+ 1.0	1.094	+ 9.5	.572	- 8.5
II. "	3.23	- 35.4	.870	- 20.5	.535	- 6.5
1919 I. "	3.36	+ 4.0	.834	- 4.1	.483	- 9.7
II. "	4.05	+ 20.5	.915	+ 9.7	.573	+ 18.6
1920 I. "	4.33	+ 6.9	.868	- 5.1	.540	- 5.8
II. "	4.27	- 1.4	.910	+ 4.8	.502	- 7.0
1921 I. "	4.62	+ 8.2	1.063	+ 16.8	.540	+ 7.6
II. "	3.55	- 23.2	.901	- 15.2	.520	- 3.7
1922 I. "	3.17	- 10.7	1.088	+ 20.8	.557	+ 7.1
II. "	3.28	+ 3.5	1.096	+ 0.7	.573	+ 2.9
1923 I. "	4.58	+ 39.6	1.094	- 0.2	.577	+ 0.7
II. "	4.18	- 8.7	.971	- 11.2	.562	- 2.6
1924 I. "	4.87	+ 16.5	1.096	+ 12.9	.577	+ 2.7
II. "	5.56	+ 14.2	1.269	+ 15.8	.610	+ 5.7

Years.	Mean Daily Areas of Promi- nences, sq. min.	Percentage Departure over Previous Half-Year.	Mean Extent of Base, min.	Percentage Departure over Previous Half-Year.	Mean Height, min.	Percentage Departure over Previous Half-Year.
1925 I. half	5.24	- 5.8	1.138	- 10.3	.588	- 3.6
II. „	6.88	+ 30.3	1.470	+ 29.2	.662	+ 12.6
1926 I. „	8.18	+ 18.9	1.615	+ 9.9	.703	+ 6.2
II. „	7.65	- 6.5	1.540	- 4.6	.657	- 6.5
1927 I. „	7.73	+ 1.0	1.744	+ 13.2	.665	+ 1.2
II. „	5.50	- 28.8	1.297	- 25.6	.593	- 10.8
1928 I. „	7.30	+ 32.7	1.947	+ 50.1	.670	+ 13.0
II. „	7.13	- 2.3	1.763	- 9.5	.663	- 1.0

The progress of the values of the mean daily areas, mean bases, and mean heights is represented in fig. 1, in which the ordinates on the left give square minutes for areas and those on the right give minutes of arc for the mean bases and mean heights of prominences. The heavy line represents the mean daily areas, the thin line the mean bases, and the broken line the mean heights. The diagram shows that, while both bases and heights exhibit sensitiveness to the changes in areas, the response made by heights is much less than the response made by the bases. To show this to better advantage, the percentage departures of the three quantities for each half-year over the previous half-year have been worked out and given in columns 3, 5, and 7 of the foregoing table and also represented in fig. 2. The departures in areas, bases, and heights are shown by the heavy, thin, and broken lines respectively. It is seen at once that, while the points on the curve for the departures in heights keep comparatively close to the X axis, those on the departure curve of the bases are in general farther above and below the X axis.

In order to discover the exact degree of relationship between the three series of data discussed here, the method of correlation was applied. There were 48 pairs of comparison in each instance. The coefficient of correlation r obtained for areas and bases was $+0.784$, with a probable error of ± 0.037 , and the coefficient r' for areas and heights was $+0.746$, with a probable error of ± 0.043 . This indicates a high degree of relationship of both bases and heights to areas, but slightly less in the case of heights. The deviations y and y' from the average of the mean bases and mean heights respectively accompanying a deviation x from the average of mean daily areas were then determined, and the results $y = .2294x$ and $y' = .0379x$ were obtained. From this we find that y is 6.05 times y' ; that is to say, any deviation in the height of prominences is on the average accompanied by 6.05 times as much deviation in their bases.

We have here an interesting result, which gives us an insight into at least some of the features characteristic of the cycles of solar activity. It is well known that corresponding to the periodicity in sunspots there is also a periodicity exhibited by the profile areas of prominences. The high coefficient of correlation between the average heights and the mean



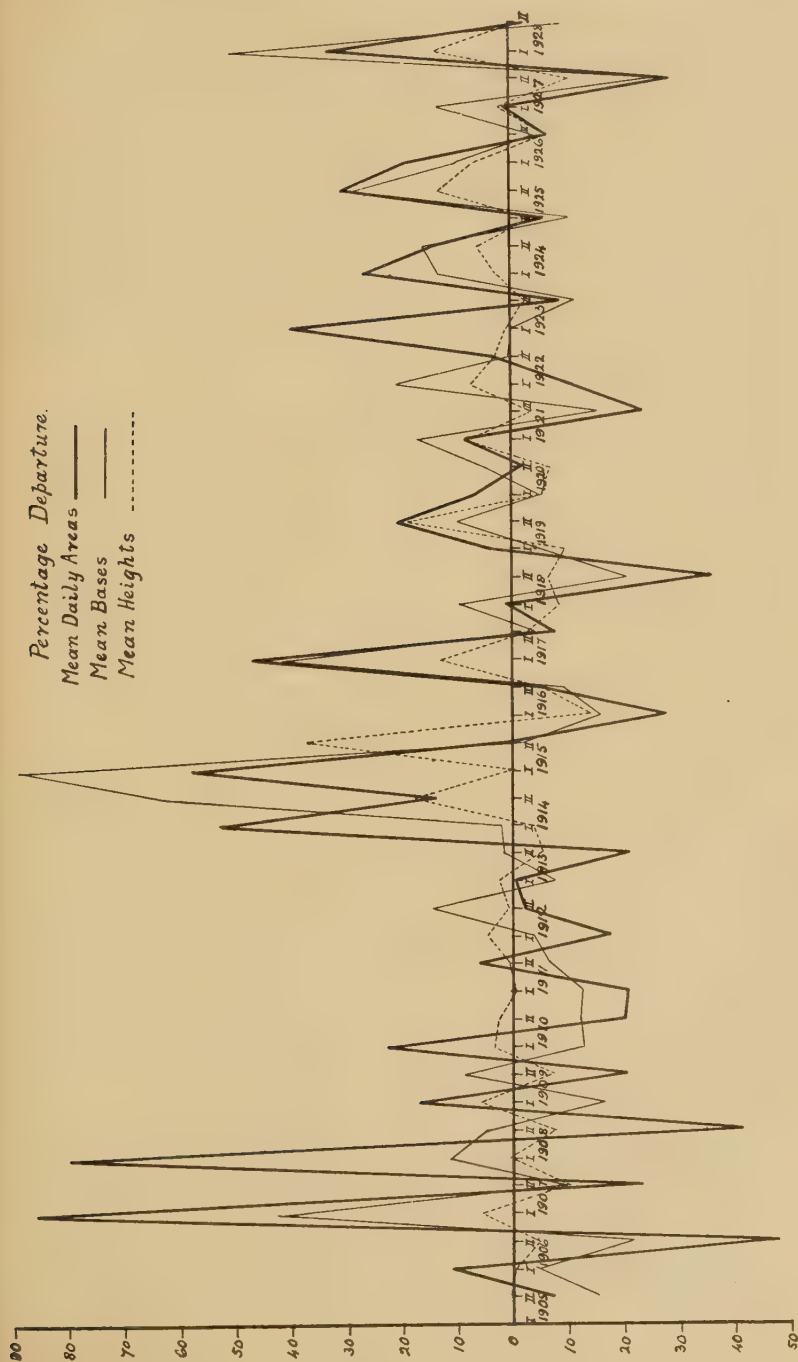


FIG. 2.

daily areas of prominences points to a similar periodicity in the average heights to which prominence matter is heaved up above the chromosphere. The natural inference seems to be that an increasing force directed outwards, whose component radial to the sun we are enabled to detect, ejects matter from the sun to greater and greater heights until the maximum activity is reached, when it begins to drop gradually to a minimum, as is evidenced by the decreasing heights of the prominences. Simultaneously with this outward projection, and with the same regular periodicity, there is also taking place, tangentially to the solar surface, a movement of matter with a force whose component along solar longitudes exhibits fluctuations which amount to six times the fluctuations in the heights of prominences. The fact that "prominence activity moves from low to high latitudes and dies out at the poles" * is probably also an indication as to the direction of the spreading movement, namely, from low latitudes towards the poles.

The idea of a pulsation naturally suggests itself to us, but a pulsation embracing two simultaneous phases. There is the heaving up and subsidence of material belonging to the outermost layers of the sun, and there is also a wavelike spreading and dying out, the latter being the more prominent feature of the two. If, as is suggested,† the sun is really a variable star with a very long period and a small range in brightness, the kind of pulsation of which we have direct evidence here seems to be the inevitable explanation for its variability. If this is so, we have secured a valuable key to the general question of stellar variability. The chances appear to be that pulsation, as Professor Eddington maintains, is the mechanism involved in certain classes of variable stars, but a slightly modified pulsation which also brings into play forces tangential to the stellar surface. From the curves in fig. 1, it would appear as though each of these pulsations in the sun consists of a series of secondary pulsations.

A glance at fig. 1 will show that, while the curve for the mean bases rises and falls correspondingly to the phases of the solar cycle, the mean height steadily increases. To a less extent this is also the case with the other two curves. It looks as though prominences once generated in a cycle do not die out completely; that is, matter ejected during a cycle is not completely reabsorbed into the body of the sun or lost in space, but a considerable residue is left behind which is being built upon during the next cycle, a process of gradual accumulation of prominences being thus indicated. Whether this is part of the evolutionary processes going on in the sun or is but a temporary state of things time alone can decide with certainty, but a separate investigation may throw some useful light on the question.

In conclusion my thanks are due to Dr. T. Royds, the Director, for the kind encouragement and valuable suggestions given to me during the investigation.

* J. Evershed, *Kodaikanal Observatory Memoirs*, 1, pt. 2, p. 64, 1917.

† *General Astronomy*, by Spencer Jones, p. 338.

The Velocity Ellipsoid, Galactic Rotation, and the Dimensions of the Stellar System. By Professor Bertil Lindblad.*(Communicated by the Secretaries.)*

According to the theory of the stellar system developed by Oort and the present writer, there should be a definite relation between the coefficient A of the differential galactic rotation * and the ratio of the two axes σ_1 and σ_2 of the velocity ellipsoid which lie in the galactic plane. If σ_1 is the axis which points toward the centre of the system, the relation in question is †

$$(1) \quad . \quad . \quad . \quad . \quad A = \omega \left[1 - \left(\frac{\sigma_2}{\sigma_1} \right)^2 \right],$$

where ω is the angular speed of rotation around the centre of the system at our distance from this centre. ω is reckoned positive in the retrograde direction. The significance of this relation is twofold. Firstly,

by co-ordinating the measured quantities A and $\frac{\sigma_2}{\sigma_1}$ for different groups of stars we may hope to get a valuable control on the theory itself; and secondly, we may in this way actually determine the quantity ω , which must otherwise be obtained by direct analysis of proper motions. The mean proper motion B in galactic longitude is connected with A and ω through the relation

$$(2) \quad . \quad . \quad . \quad . \quad B = A - \omega.$$

The value of B derived directly from the proper motions has to be compared with the value given by (2).

We have tacitly assumed above that the directions of the axes of the velocity ellipsoid are in agreement with the direction towards the centre, which is indicated by the effect of differential rotation. One of the two axes in the galactic plane should point towards the centre of rotation. From the phenomenon of the "asymmetrical drift" of high stellar velocities we deduce that ω should be positive, and if we interpret the differential rotation with respect to a centre about the galactic longitude 325° we get positive values of A . The bigger axis, or the vertex line, should then point towards the centre. This is a primary condition which is to be fulfilled, and we shall have occasion to examine below how the vertex line and the centre line actually fit for different groups. According to the abstract theory the third axis of the ellipsoid, at right angles to the galactic plane, should be equal in length to the longer axis situated in that plane, and thus the velocity ellipsoid should be a spheroid with the small axis in the galactic plane and at right

* Oort, *B.A.N.*, Nos. 120, 132, 133, 1927.

† Lindblad, *M.N.R.A.S.*, 87, 562, 1927; *Arkiv för Matem., Astronomi och Fysik*, 20 A, No. 17, 1927; 21 A, No. 3, 1928; 21 A, No. 15, 1929. Oort, *B.A.N.*, No. 159, 1928.

angles to the centre line. In the neighbourhood of the galactic plane, however, and especially for groups of strong concentration toward this plane, the motion will largely be of a two-dimensional character, so that the components of velocity parallel to the galactic plane have only a very slight statistical connection with the third component at right angles to the plane. In fact, when the star belongs to a "sub-system" of high speed of rotation, *i.e.* with a nearly circular motion around the galactic centre, the motion of oscillation in the third direction and the motion along the projection of the radius vector from the centre on the galactic plane are small and nearly independent of one another. It is usually found in the statistics of stellar radial velocities that the third axis, at right angles to the galactic plane, is intermediate in size between the two other axes, and often considerably smaller than the axis of the vertex line.

On account of what has just been said above we may still suppose that the two-dimensional motion defined by the velocity components in the galactic plane can be treated as a steady phenomenon. The ratio of the two axes of the velocity ellipsoid which lie in the galactic plane should then still conform to equation (1). On the other hand, this presupposes that the velocity distribution of the stars around the sun is determined by an infinite number of independent streams, each one of infinitesimal mass. It is possible, however, that the observed surplus of velocity dispersion in the galactic plane, in comparison with the dispersion in the third direction, is due to some stream or streams of finite mass which at the present moment are transverse to our point of the system and thus exert a finite disturbance on the velocity figure. In such a case we must expect other discrepancies from the ideal state to go parallel with a deviation from unity of the ratio between the third and the first axis. In other words, the occurrence of streams of finite mass will make it impossible to apply strictly the "law of great numbers" to the velocity distribution, and will cause accidental deviations from the theoretical velocity figure. The streams in question may be supposed to obey the same laws of chance as the infinitesimal ones; it is only the small number of such streams of great mass which disturb the statistical balance. We have every reason to accept this hypothesis, as we know at least one such stream (Ursa Major) which may be of considerable importance for the velocity distribution around the sun. In the case of the Ursa Major stream in a wider sense, we know from Strömberg's* work that its domain is very vast and that it plays an important part in the velocity distribution of the A-type stars. Some strong support for the hypothesis in question will be found below in the discussion of our empirical results. We get in this way, on the basis of a dynamical theory, a sort of compromise between the ellipsoidal theory and Kapteyn's theory of special streams.

The material of the present investigation consists of all radial velocities published up to the year 1929, with the exclusion of a few of the oldest lists. It was collected in a card catalogue comprising data for about 4600 stars. The radial velocities from different observatories

* *Mount Wilson Contr.*, No. 257, 1922; *Astrophys. Journal*, 57, 77, 1923.

were reduced to the Lick system by forming the residuals for the stars in common with Lick. The spectral types B, A, F, G, K, M were treated separately. The system of corrections arrived at in this way is given in Table 1. The numbers in parentheses are the respective numbers of stars.

TABLE 1.

Systematic Corrections.

	B.	A.	F.	G.	K.	M.
Allegheny	..	+4.2 (21)	+5.0 (5)			
Ann Arbor	-0.3 (3)	-0.3 (9)				
Bonn	..	..	-0.2 (37)	-1.0 (51)	-2.2 (118)	-2.8 (11)
Cape	+1.1 (3)	+0.7 (6)	+1.0 (62)	-0.2 (92)	-0.3 (248)	-0.2 (44)
Mt. Wilson	-1.1 (60)	-0.3 (60)	+0.3 (64)	-0.2 (93)	-0.8 (73)	-0.7 (40)
Victoria	+4.6 (10)	+2.6 (48)	+1.5 (29)	+0.6 (39)	+0.2 (44)	+1.4 (14)
Yerkes	-0.8 (189)	+0.8 (3)				

The aim of the investigation is to study the effect of differential rotation and the velocity ellipsoid for the same groups of stars. We must therefore form groups of sufficiently large mean distances $\bar{r}$ to allow a determination of the coefficient $\bar{r}A$, and at the same time containing a sufficiently large number of stars for a good determination of the velocity ellipsoids. For the B stars two groups were formed containing the types B₀-B₇ and B₈-B₉ respectively. The stars of later types were divided according to proper motion into sub-groups with $\mu < 0''.040$, $0''.040 < \mu < 0''.160$, $\mu > 0''.160$ respectively. The A stars were treated separately, but for the types F-K rather extensive preliminary computations made it evident that the characteristics of the velocity distribution were sufficiently similar to allow these types to be taken together within all the three proper motion groups. Later types were not considered.

With regard to the methods for computing the differential rotation and velocity ellipsoids much care has been taken to avoid the effects of systematic errors. In the case of the former effect we must especially be on our guard against errors arising from a poorly determined apex motion and an uneven distribution of the stars along the galactic circle. It is further easy to see that a division with regard to proper motion may introduce a term in the radial velocities which varies with the galactic longitude. This effect is illustrated in fig. 1. Let OR be the radius vector from the observer to the area in the sky considered. Let E be a section of the velocity ellipsoid and PR the direction of the parallactic motion. A selection of small proper motions will mean a selection of short velocity vectors counted from P, and the effect of this must be to shift the mean radial velocity toward the observer, *i.e.* in the negative direction. However, this effect, as well as the entire effect of the parallactic motion, is completely eliminated, if we take the unweighted mean between the mean radial velocities of two opposite

areas in the sky. This mean has to be identified with the rotation effect, which can be written

$$(3) \quad \dots \quad \bar{r}A \cos^2 b \sin 2(l - l_0),$$

where l and b are the galactic co-ordinates, and l_0 is the galactic longitude of the centre.

The sky was divided into 48 equal "galactic" areas according to Charlier's scheme. For the rotation effect only the 24 areas around the galactic equator were used, because in this case the areas of higher galactic latitude have very little weight, and their inclusion would rather tend to make the material non-homogeneous. Opposite areas

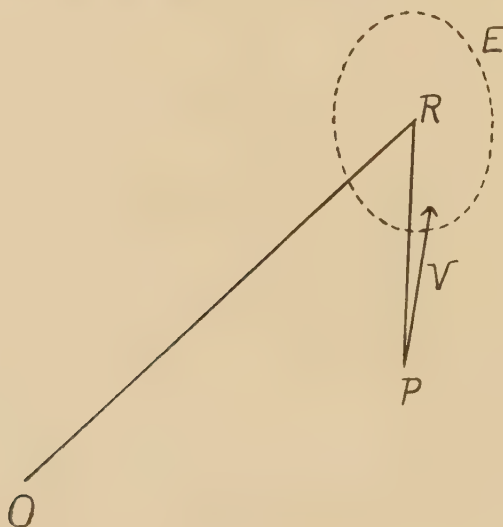


FIG. 1.

were combined with equal weight in the way just described, and a weight was assigned to their mean equal to the number of stars in the poorer of the two regions. For a given central longitude between 0° and 180° there are two such pairs of areas, and a weighted mean was taken between these. We get in this way six equations of condition for determining the three unknown quantities

$$K, \quad x = \bar{r}A \cos^2 b \cos 2l_0, \quad y = -\bar{r}A \cos^2 b \sin 2l_0.$$

The weights of the equations of condition were the combined weights of the two pairs.

For computing the parallactic motion Charlier's* method was adopted. With the values of apex and solar motion found in this

* "The Motion and the Distribution of the Stars," *Memoirs of the University of California*, vol. vii. (1926).

way the parallactic motion in the radial velocity was computed for each area, and the rotation effect was added. The residuals of the observed velocities were formed within each area, and the average residual without regard to sign was computed. The weighted mean of opposite areas was then taken. In order to get the most trustworthy determination of the orientation of the ellipsoid with regard to galactic longitude, it was further assumed in the final calculations that the ellipsoid is symmetrically arranged with regard to the galactic plane, so that the weighted means could be taken for four areas of the same mean longitude and of the same numerical value of the galactic latitude. In the case of the B stars this process of smoothing was continued one step further, in that for all areas of numerical value of the latitude exceeding 30° a common general mean was taken. The dispersion of the radial velocities was formed by multiplying the average residuals by the factor 1.253, and the computation of the ellipsoid was then performed in the way outlined by Charlier in his work cited. As the vertex line then fell exactly in the galactic plane, the computation was of course much simplified. It should be mentioned here that preliminary calculations without making the assumption of symmetry with respect to the galactic plane fully justify the simplified procedure and, in fact, indicate that we get in this way more reliable results without imposing unnatural restrictions.

A few stars were excluded on account of very large residual velocities. In the Bo-B7 group the members of the h , χ Persei cluster were omitted.

Some further remarks concerning the computations for different groups are given below.

Bo-B7.—In addition to the calculations described above, the parallactic motion was computed from radial velocities as well as from proper motions, giving weights to each area equal to the respective numbers of stars. To the proper motions were applied the corrections derived by Boss* and by Raymond.† The following constants of the solar motion with regard to this group of stars were derived. The solar speed and the K term from the radial velocities with their mean errors are expressed in km./sec.

	Apex.		Solar Speed.	K.	No.
	<i>l.</i>	<i>b.</i>			
R.V.	$31^\circ \pm 2$	$13^\circ \pm 5$	22.25 ± 0.8	4.54 ± 0.6	445
P.M.	$18^\circ \pm 5$	$20^\circ \pm 2$	$0''.0243 \pm 19$		

Accordingly we get the mean parallax

$$\bar{\pi} = 0''.0052.$$

The mean apparent magnitude is $\bar{m} = 4.74$, and the mean numerical value of the galactic latitude for determining the rotation effect is $b = 10^\circ.9$.

* *Preliminary General Catalogue*, p. xxviii, 1910.

† *Astronomical Journal*, No. 857, 1926.

B8-B9.—The analogous results for this group are as follows :—

	Apex.		Solar Speed.	K.	No.
	<i>l.</i>	<i>b.</i>			
R.V.	33°±5	23°±8	21.76±1.7	2.79±1.0	250
P.M.	19°±6	18°±2	0".0355±28		

$$\bar{\pi} = 0''.0077, \\ \bar{m} = 5.07, \quad b = 12^\circ.9.$$

A-K.—For the stars in the areas adjacent to the galactic equator the mean apparent magnitude and the mean proper motion were determined. The mean parallax was then found from van Rhijn's* tables. The results for the differential rotation are only given for the first proper motion group. The values found for the other two groups must be considered illusory, on account of the smaller mean distances of the stars, and also because of the high dispersion of the velocities which increases the mean error of the result. The three proper motion groups are indicated in order by I, II, III respectively.

The immediate results of the computations for differential rotation and the velocity ellipsoids are given in Tables 2 and 3. For all groups in Table 2 the mean distance was formed from the mean parallax by means of the formula

$$\bar{r} = \frac{1.27}{\bar{\pi}},$$

corresponding to a dispersion in the modulus $m-M$ slightly exceeding one magnitude, and in the case of the types A-K in accordance with the distribution of π round $\bar{\pi}$ derived by van Rhijn.

TABLE 2.

Differential Rotation.

Group.	K.	<i>x.</i>	<i>y.</i>	<i>l</i> ₀ .	A.	$\bar{m}$.	$\bar{\mu}$.	$\bar{\pi}$.	$\bar{M}$.
Bo-B7	+4.3±0.3	+0.5±0.3	+1.4±0.4	324°	+0.006	4.8	..	0''.0052	-1.9
B8-B9	+3.6±0.3	+1.3±0.5	+1.6±0.5	334°	+0.013	5.1	..	0''.0077	-0.7
A, I	-1.0±2.4	+0.7±3.2	+4.9±3.8	319°	+0.033	5.3	0''.022	0''.0078	-0.5
F-K, I	-2.5±0.7	+3.3±1.0	+1.8±1.0	346°	+0.021	5.4	0''.020	0''.0067	-0.7

The results of Table 2 confirm entirely previous results derived by Oort, Schilt, Plaskett, and Dyson concerning the existence of a term due to "differential rotation." The solution of the equations of condition for the A stars gives, however, very large mean errors of x and y , due to a couple of large residuals. A weighted mean of x and y for the groups gives

$$l_0 = 332^\circ \pm 5^\circ.$$

This value of l_0 is quite in agreement with a position of the galactic centre in a galactic longitude around $l = 325^\circ$, as indicated by the

* *Groningen Publications*, 34, 1923.

central point of the distribution of globular clusters (Bohlin, Hinks, Shapley), by the apparent distribution of faint stars (Seares, Kreiken), by the phenomenon of the asymmetrical drift of high stellar velocities, and lastly by the results just mentioned concerning the differential rotation.

TABLE 3.
Velocity Ellipsoids.

Group.	Vertex.	σ_1 .	σ_2 .	σ_3 .	$\frac{\sigma_2}{\sigma_1}$.	$\frac{\sigma_3}{\sigma_1}$.	N.
		km.	km.	km.			
B0-B7	289°	10.9	9.8	4.5	0.90	0.41	445
B8-B9	275°	12.4	10.5	11.4	0.85	0.92	250
A, I	21°	18.6	12.4	3.9	0.66	0.21	304
F-K, I	335°	17.5	13.6	16.5	0.78	0.94	714
A, II	356°	19.1	8.4	10.0	0.44	0.52	391
F-K, II	342°	27.7	19.4	16.9	0.70	0.61	857
F-K, III	349°	36.3	21.4	25.5	0.59	0.70	825

After this preliminary discussion of Table 2 we turn our attention to the results for the velocity ellipsoids in Table 3. It is evident that a very sudden change in the velocity surface takes place at the transition from the B to the A stars. The direction of the vertex line for the B stars points towards considerably lower values of l than those indicated by the values of l_0 from the differential rotation. The velocity surface for the B stars has, however, so nearly circular symmetry in the galactic plane that the vertex direction can only be rather uncertainly defined. The tendency of these stars to form vast local clusters probably accounts for a great deal of the deviations from the ideal velocity surface according to the theory. A considerable part of the observed velocities of these stars may correspond to relative motions of the clusters, in which the stars are more or less definitely enclosed with small relative motions.

More tangible results are no doubt given by the types A-K. The vertex line points here, especially for the A stars, toward somewhat too high galactic longitudes, if we assume the longitude of the centre to lie in the neighbourhood of $l = 325^\circ$. The best agreement is shown by the first F-K group, and it is evident that this group agrees also in another way very well with our theory, as the ratio $\frac{\sigma_3}{\sigma_1}$ is very nearly equal to unity. It lies, then, near at hand to compare the deviation of the vertex line with the deviation of the ratio $\frac{\sigma_3}{\sigma_1}$ from its theoretical value. It is evident that a decided correlation seems to exist between these two discrepancies, which are, for instance, both most marked in the first A-type group. Fig. 2 gives a graphical representation of the relation between the longitude of the vertex and $\frac{\sigma_3}{\sigma_1}$. The representative

points are distributed fairly well along a straight line. The best straight line drawn through the points intersects the line $\frac{\sigma_3}{\sigma_1} = 1.0$ at the longitude

$$l = 324^\circ.$$

By chance this value agrees almost exactly with our ideas regarding the longitude of the centre. Without laying weight on this exact coincidence we may say that this result indicates that the deviations of the observed velocity surface from the theoretical one are correlated with one another, and that they reach a definite maximum in the A type, where the velocity surface is very much elongated in a certain direction of the galactic plane. This direction coincides fairly well with the vertex of the Ursa Major stream, which according to Rasmuson has

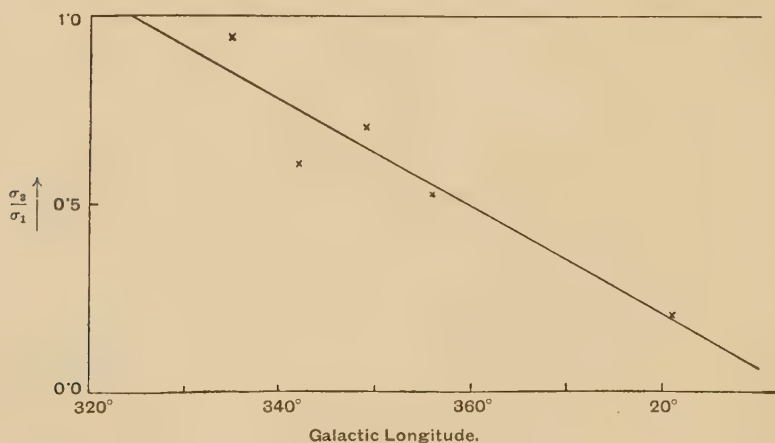


FIG. 2.

the co-ordinates $l = 0^\circ$, $b = -8^\circ$. That the stream in question is responsible for a great deal of the characteristics of the velocity surface for the A-type stars has already been emphasized by Strömberg on the basis of a study of the space motions of these stars, and is further indicated here by the abnormally low value of the solar motion. By applying Charlier's formulæ* to the various groups we find the data of Table 4. U_0 , V_0 , W_0 are the components of the parallactic motion in the galactic system. The U -axis points toward galactic longitude 0° and the W -axis toward the galactic pole. For the B stars the results of a more elaborate computation of the apex motion have been given above. The areas of high galactic latitude are too poor for these groups to allow a calculation in which equal weights are given to different areas.

It is evident that the centre of the velocity surface of the first group of A stars is displaced in about the direction of the Ursa Major stream (U -axis) relative to the centres of motion of other groups. According

* *Loc. cit.*, p. 55.

to the foregoing discussion it seems as if the influence of the stream were extended also to the nearer stars of types F-K, while it is nearly absent from the remotest group, F-K, I.

TABLE 4.
Apex Motion.

Group.	K .	U_0 .	V_0 .	W_0 .	l .	b .	S .
A, I	-0.4	-9.1	-4.7	-9.3	27°	+42°	13.8
F-K, I	-0.9	-13.7	-6.4	-8.5	25°	+29°	17.3
A, II	+0.6	-18.1	-2.5	-6.7	8°	+20°	19.4
F-K, II	-0.9	-14.6	-5.6	-6.0	21°	+21°	16.7
F-K, III	+0.9	-22.8	-7.9	-3.5	19°	+8°	24.4

If we accept as galactic longitude of the centre $l_0 = 324^\circ$, we may recompute the coefficient A for the different groups and then also for comparison compute the ratio $\frac{\sigma_2'}{\sigma_1'}$ of the dispersions for the directions $l = 54^\circ$ and $l = 324^\circ$ in the galactic plane, *i.e.* for the ideal directions of the axes of the velocity ellipsoid. For the latter computation we use the data of the velocity ellipsoids in Table 3. We then get the results which are given in Table 5.

TABLE 5.

Group.	K .	A .	$\frac{\sigma_2'}{\sigma_1'}$.	N .
Bo-B7	+4.3	+0.006±2	0.97	445
B8-B9	+3.7	+0.012±3	1.02	250
A, I	-1.2	+0.034±20	1.17	304
F-K, I	-2.2	+0.015±9	0.80	714
A, II	..	..	0.74	391
F-K, II	..	..	0.78	857
F-K, III	..	..	0.74	825

The size of the mean errors of A scarcely allows us to accept a variation of this quantity with the spectral type, though the small value for Bo-B7 may possibly be taken as a consequence of the large value of $\frac{\sigma_2'}{\sigma_1'}$. On the other hand, the mean errors given in the table are probably no safe indicators of the relative reliability of the results. In order to avoid this difficulty, I have made a final computation for the groups Bo-B7, B8-B9, A, I, F-K, I, taken together. For each area the weighted mean of the four groups was taken after having corrected for the K -term according to the values of Table 5. We then proceed in the same way as described above for a single group. It may be remarked that by simply adding together the radial velocities for opposite regions of the sky with their signs, as done here, the influence of a local stream

will be eliminated, at the same time as the parallactic motion, as soon as the stream is uniform enough to enter into the two areas in the same abundance, in which case it affects the mean radial velocities of the two regions by equal amounts but with opposite signs.

We get in this way our final value

$$A = +0.015 \pm 0.005 \text{ km. per sec. per parsec.}$$

for $l_0 = 324^\circ$. Without any assumption regarding l_0 we get

$$A = +0.017 \pm 0.004, \quad l_0 = 338^\circ \pm 8.$$

Thus the final value of A agrees with the value of A found for the group F-K, I, separately. We remember that this group gives a velocity ellipsoid in close agreement with the theoretical demands. In accordance with our previous discussion, we have reasons to suppose that the values of $\frac{\sigma_2'}{\sigma_1}$ for the B and A stars may be seriously affected by accidental streamings, and we are therefore obliged to adopt the value 0.80 of the group F-K, I. This value is further supported by the results for the nearer stars, groups A, II, and F-K, II and III, as shown in Table 5. The general weighted mean of *all* the groups in this table gives 0.84, and excluding the group A, I, we get 0.81. We can therefore scarcely commit any serious error by assuming as our final value

$$\frac{\sigma_2'}{\sigma_1} = 0.80.$$

Identifying $\frac{\sigma_2'}{\sigma_1}$ with $\frac{\sigma_2}{\sigma_1}$ we get, according to equations (1) and (2),

$$\omega = +0.042, \quad B = -0.027.$$

Our value of A agrees well with Oort's value $A = +0.019 \pm 0.003$, and with Plaskett's * value for B and O stars, $A = +0.0155 \pm 0.0007$.

By division with the factor 4.74 we get our values of A , ω , and B expressed in seconds of arc as follows:

$$A = +0''.0032, \quad \omega = +0''.0088, \quad B = -0''.0057.$$

It is remarkable how well the value of B agrees with the results derived directly from the proper motions. Charlier,† who seems to have been the first to determine this quantity with any precision, has derived the value $-0''.0024$, to which he attaches the remark that it must be increased numerically by the still undetermined precession of the invariable plane due to the action of the stars on our planetary system. Oort adopts after a careful reduction as the best value $-0''.0050$. It is of special interest to compare further our values of the coefficients A and B , which depend on the radial velocities and on the form of the velocity ellipsoid, with the values of these coefficients determined for

* *M.N.R.A.S.*, **88**, 402, 1928.

† *Loc. cit.*, p. 32.

nearly the same group of stars, but entirely from the proper motions. Oort derives for a collected group of B, c, O, N, and δ Cephei stars,

$$A = + 0''.0024 \pm 16, \quad B = - 0''.0050 \pm 11, \quad l_0 = 326^\circ \pm 17.$$

Quite recently Dyson * has found for Bo-Ao stars

$$A = + 0''.0025, \quad B = - 0''.0046, \quad l_0 = 327^\circ.$$

It is evident that these results are quite in accordance with our values obtained above, which fact speaks strongly in favour of the reality of the effects in question, as the methods used are radically different. It would seem also that the precession of the invariable plane predicted by Charlier is only very slight.

We shall sum up briefly the consequences of the values of the characteristic quantities derived above for our conceptions regarding the size and mass of the greater stellar system. ω gives at once the period of a revolution of the system at our distance from the centre to be

$$P = 150 \text{ million years.}$$

In order to get the dimensions of the system we must estimate the size of the linear rotational motion, Θ_0 , of our groups of stars. If we accept for this quantity 270 km./sec., according to the apex determinations for the globular clusters performed by Lundmark and Strömberg, we get for our distance from the centre

$$\varpi = 6500 \text{ parsec.}$$

If we make the assumption that the distribution of mass in the system can be described with a certain approximation as determined by a central nucleus of mass M_1 , in addition to a very flattened spheroid (or a series of such spheroids) with the radius ϖ_1 in the galactic plane and with the mass M_2 , the gravitational force inside the spheroid and near the galactic plane will be a sum of two forces, one inversely proportional to the square of the distance from the centre, and one directly proportional to this distance. It is then easy to show that with the system of notation and with the units used above we have the following formulæ for determining M_1 and M_2 :

$$M_1 = 0.617 \frac{A}{\omega} \varpi \Theta_0^2 \cdot 10^{36} \text{ gr.}$$

$$\frac{M_2}{M_1} = \frac{1}{\pi} \left(\frac{\varpi_1}{\varpi} \right)^3 \left(\frac{\omega}{A} - \frac{4}{3} \right).$$

The ratio $\frac{\varpi_1}{\varpi}$ can be estimated from the phenomenon of the asymmetrical drift. In a recent paper † I have obtained a value about 3/4. We then get immediately

$$M_1 = 109 \cdot 10^{42} \text{ gr.}, \quad M_2 = 119 \cdot 10^{42} \text{ gr.}$$

* *M.N.R.A.S.*, **90**, 239, 1930.

† *A.N.*, **236**, 181, 1929.

The combined mass is then

$$M_1 + M_2 = 228 \cdot 10^{42} \text{ gr.} = 11 \cdot 10^{10} \text{ solar masses.}$$

This value of the total mass does not deviate very sensibly from the value $18 \cdot 10^{10}$ solar masses which was derived in my first paper on the present subject.*

Summary and Conclusions.

The present treatment of the problem of stellar motions presupposes that the main features of the distribution of stellar motions around the sun can be treated as consequences of an approximately steady motion of the entire stellar system.

The fundamental fact on which the theory has been built up, as far as the present author is concerned, is the phenomenon of the asymmetrical drift of the velocity centres of groups with increasing dispersion, a phenomenon which has only been touched upon lightly in the present paper, but has been more intimately treated in previous papers.† This phenomenon shows conclusively that the cause of the ellipsoidal distribution of stellar velocities cannot be sought for in a "local system," because the centres of ellipsoids of approximately parallel axes do not coincide, but show a systematic motion with respect to one another (in a direction which points pretty nearly in the direction of the supposed centre of the "local system" or "local cluster"). This effect of the asymmetrical drift is shown here in the large solar motion of the last group, F-K, III, of Table 4.

It is tempting to associate instead the "local cluster" with the disturbances of the form of some of the velocity ellipsoids treated above, namely, those of the B stars and the bright A stars. The "local cluster" should then consist chiefly of the members of vast streams of these stars, of which the most important ones are no doubt indicated by the huge agglomerations of B stars in the southern sky around the Carina region, and in the case of the A stars by the Ursa Major streaming. It is very probable that these streams contain also a great number of stars of smaller absolute luminosities than the B and A stars. It should be mentioned here that H. Mineur ‡ in an interesting paper has found an effect in the second order terms of the mean radial velocities, developed in spherical functions of the angular co-ordinates, which can be interpreted as an effect of a differential rotation of a quite special character around a centre nearly coinciding with Charlier's centre of the near B stars. It is, however, difficult to see why there should be any immediate connection between this special effect, which vanishes in the galactic plane, and Oort's effect of differential rotation. Mineur makes, however, the obvious remark that the latter effect admits a change of 90° in l_0 , if we change at the same time the sign of the coefficient A , which would bring the value of l_0 in near agreement with Charlier's centre. He tries to dissect the material for which

* *Arkiv för Matem., Astronomi och Fysik*, 19 A, No. 21, 1925.

† *Loc. cit.*

‡ *C.R.*, 188, pp. 236, 1086, 1378, 1929; *Bull. Astr.*, 5, Fasc. 8, 1929.

determinations of differential rotation according to Oort's scheme exist into two classes, one class of distant stars with rotation around the centre at $l = 325^\circ$, the other a class of near stars rotating around a centre at $l = 240^\circ$. The good agreement found in the present paper between the effect in the F-K stars, which do not show signs of belonging to a local cluster, and the bright B stars (especially Dyson's results from the proper motions) makes, however, such a dissection rather doubtful. The general agreement of the effect for the different groups, of different mean distances, studied by Oort and Plaskett, would likewise be very difficult to understand. For the more distant groups the mean distance widely surpasses the distance of Charlier's centre. One may draw attention in this connection to the wide difference in centre of motion between the B stars and the bright A stars, and between both these types and the F-K stars, which differences appear to give support to a "stream theory" of the local cluster in the way proposed above. That in this way the conditions of the motions in the "local cluster" will be utterly complicated and difficult to interpret is evident. The orbits of the streams in relation to the big system will only differ slightly from circular motions around the centre, but in their relative motions the streams might have a perceptible influence on one another. Still, it does not seem necessary to give up at present the idea that the gravitational field governing the relative motions of these streams is in the main the regular field of the great stellar system. In fact, simple considerations of mass and space frequency indicate that the gravitational field is determined by the stars of small masses, and that the giant stars of low velocity dispersion which show strong tendencies of clustering have only a slight influence on the field.

It seems that the following general explanation of the evolution of these streams might be ventured. As explained in a previous paper cited above, the "encounters" between individual stars will tend to move slowly the bulk of heavy stars towards the centre of the stellar system, and for outer regions of the system toward the galactic plane. We may say the heavy stars will tend to form a very flattened sub-system with small dispersion of the relative velocities (where the individuals move in nearly circular orbits around the centre of the stellar system). On account of the small relative velocities this sub-system is not stable as a thin uniform layer but tends to break up into local clusters which acquire by mutual disturbances slow relative motions parallel to the galactic plane, *i.e.* they acquire the character of streams.

We have tried here in a study of the radial velocities of the "ordinary" spectral types to extract the effects which according to the general theory of the steady state are of importance for fixing the fundamental properties of the system. The results, which are based entirely on the radial velocities (differential rotation and velocity ellipsoid) show good agreement with the results from the proper motions.

The dimensions of the system derived are of the same order of magnitude as those of Shapley's system based on an analysis of the distribution of globular clusters. The uncertainty of several quantities

entering in our calculation forbids us to widen the discussion on this point. It may only be remarked that in the case of the system of clusters the probable dynamical centre should be sought for at the point of densest agglomeration of the clusters, and not in the geometrical centre defined by the mean co-ordinates of the clusters. According to the general theory these clusters should rather be considered as a system of satellites, of which individual members may depart to extremely great distances from the system itself.

Observatory, Stockholm :
1930 March.

Some Remarks concerning the Galactic Rotation.

By Henri Mineur.

(Communicated by the Secretaries.)

I. Introduction.

1. The purpose of this paper is to sum up my researches on the subject of the galactic rotation published in the *Comptes Rendus de l'Académie des Sciences de Paris*, and in the *Bulletin Astronomique* and in the *Bulletin de l'Observatoire de Lyon* (1).

2. Oort, Plaskett, and all the astronomers (2) who have studied this question have represented the mean radial velocity (v) of the stars at the galactic longitude (G) by a trigonometrical series :

$$v = K + \alpha_1 \sin G + \beta_1 \cos G + \alpha_2 \sin 2G + \beta_2 \cos 2G + \alpha_3 \sin 3G + \beta_3 \cos 3G + \dots$$

They have calculated the numerical values of the coefficients α_i and β_i .

The terms $\alpha_2 \sin 2G + \beta_2 \cos 2G$ stand for the galactic rotation ; if G_0 is the longitude of the axis, these terms should have the following form :

$$A \sin 2(G - G_0).$$

So we have :

$$(1) \quad \tan 2G_0 = -\frac{\beta_2}{\alpha_2}, \quad A = -\frac{\beta_2}{\sin 2G_0}.$$

All the authors who have tried to determine the value of G_0 from the values of α_2 and β_2 have found values of 330° approximately. This value is about the same as the longitude of the galactic centre as it has been found by Shapley. And so it has been concluded that the galaxy revolves around an axis situated in the direction of Sagittarius (longitude 330°).

Here I must direct attention to the following point. The system (1) admits four solutions :

$G_0.$	$G_0 + 90^\circ.$	$G_0 + 180^\circ.$	$G_0 + 270^\circ.$
A	$-A$	A	$-A$

Until now the solution $G_0 = 330^\circ$

has been favoured because the longitude of the galactic centre is 325° . But we may also choose the solution

$$G_0 = 240^\circ,$$

for the local cloud has its centre at this longitude.

3. It is proved (3) that the galaxy is an immense mass of stars whose centre is at the longitude $G_0 = 325^\circ$. But numerous studies prove that the sun is inside a star cloud which constitutes a minor condensation in the galaxy; this local cloud, which includes many millions of stars, has its centre at longitude 243° .

So we may well ask ourselves whether the results obtained up to the present stand for a galactic rotation around an axis situated at longitude 325° or at longitude 243° . I shall demonstrate my reasons for thinking the second hypothesis preferable.

The terms in $2G$ stand for the rotation of the local cloud.

II. Statistical Study of G_0 .

4. More precisely, the centre of the galaxy is at the longitude 325° ; that of the local cloud at the longitude 243° . Now,

$$243^\circ + 90^\circ = 333^\circ.$$

So it all comes to show that the values obtained for G_0 are nearer to 333° than to 325° .

We have divided the values of G_0 into two categories.

First category: Values of G_0 obtained with stars nearer than 600 parsecs to the sun and situated inside the local cloud.

Second category: Values of G_0 obtained with stars situated at more than 800 parsecs from the sun, *i.e.* outside the local cloud.

We have computed for each category the number of values of G_0 which fall in given intervals of longitude:

G.	300°.	302°·5.	305°.	307°·5.	310°.	312°·5.	315°.	317°·5.	320°.	322°·5.	325°.	327°·5.	330°.
First category (local cloud)	1	1	0	1	3	2	1	1	0	1	8	8	5
Second category (galaxy)	0	0	0	1	1	0	0	0	2	2	4	5	1

G.	332°·5.	335°.	337°·5.	340°.	342°·5.	345°.	347°·5.	350°.	352°·5.	355°.	357°·5.	360°.	2°·5.
First category (local cloud)	8	7	4	5	4	3	4	2	1	1	2	1	0
Second category (galaxy)	0	0	0	0	0	0	0	0	0	0	0	0	0

These two frequency curves of G_0 are represented by the figure, p. 518.

It appears that the mean value of G_0 of the first category is $332^\circ\cdot7$ (local cloud), whilst the mean value of G_0 of the second category (galaxy) is $323^\circ\cdot2$.

We conclude that (1) the nearest stars revolve around an axis whose longitude is :

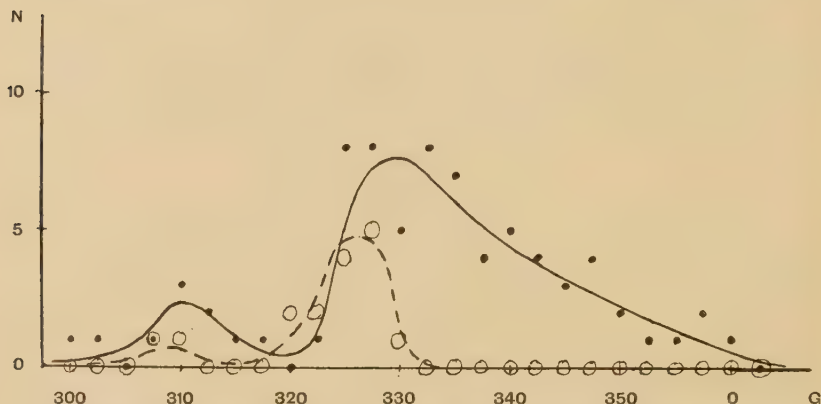
$$G_0 = 333^\circ + K \cdot 90^\circ \quad (K = \text{a whole number})$$

and we take $K = -1$ as the value of $G_0 = 243^\circ$ coincides with the longitude of the centre of the nearest stars ;

(2) the distant stars revolve around an axis whose longitude is

$$G_0 = 323^\circ + K' \cdot 90^\circ \quad (K' = \text{a whole number})$$

and we take $K' = 0$ so as to fall in with the longitude of the galactic centre.



x axis = longitudes G . y axis = No. of values found by different authors (Oort, Plaskett, Schilt . . .) for longitude of axis of rotation comprised between $G - 2^\circ.5$ and $G + 2^\circ.5$. The points and full curve refer to the stars situated in the local cloud ($r < 600$ parsecs), the circles and dotted curve refer to the remote stars ($r > 800$ parsecs).

A glance at the curves shows that, statistically, the nearest stars give for the longitude of the axis the value $332^\circ.7$, somewhat greater than the value $323^\circ.2$ given by the remote stars.

III. General Study of the Radial Velocities of the Stars.

5. Oort and Plaskett have only considered the radial velocities of stars situated in the galactic plane. In a work I mentioned before I have studied the question of the radial velocities of stars situated in every direction.

6. Let us begin with the following theoretical considerations.

Let us suppose that the sun is localised in a revolving cloud at a given distance, r_0 , from the axis and at a given distance, Z_0 , from the plane of symmetry of the cloud, this being actually the position of the sun in the local cloud, with

$$r_0 = 80 \text{ parsecs}, \quad Z_0 = 60 \text{ parsecs above the plane of symmetry.}$$

Let us suppose again that the angular velocity Ω of a star is dependent on its distance from the axis of rotation ωu and on its distance from

the plane of symmetry of the cloud. The result will be a differential rotation of the stars of the cloud similar to the phenomenon found out by Oort. But here the phenomenon is more complex, as the differential rotation depends no longer on one variable only, as Oort has admitted, but on two, namely, the distance from the axis and the distance from the plane.

Let us take axes, with the sun as centre, as follows: The axis OZ parallel to the axis of rotation ωu , the axis OX being the perpendicular drawn from O to axis ωu .

It is to be remembered that XOY is not a plane of symmetry of the cloud, but is the plane parallel to the plane of symmetry of the cloud passing through the sun.

So let us suppose that the angular velocity Ω of a star depends on its distance ρ from ωu and on its distance z from the plane of symmetry, so that

$$\Omega = f(\rho, z).$$

Taking v as the mean radial velocity of the stars situated at a given distance r from the sun in the direction whose direction cosines are α, β, γ , let us develop v into a series of spherical functions:

$$v = K + \phi_1(\alpha, \beta, \gamma) + \phi_2(\alpha, \beta, \gamma) + \dots$$

ϕ_i meaning the terms of degree i .

It is easy to see that (1) ϕ_1 stands for the motion of translation of the sun toward the apex; (2) ϕ_2 stands for the rotation of the stars. The cone $\phi_2 = 0$ degenerates into two planes, P_1 and P_2 .

One of these planes, P_1 , is XOZ; it is the galactic meridian which passes through the axis of rotation ωu .

The other plane depends on the function f . (a) If f does not depend on z , this plane P_2 is YOZ, the galactic meridian perpendicular to the first plane P_1 . (b) If f does not depend on ρ , this plane P_2 is the galactic plane XOY. (c) If f depends on both ρ and z , the plane P_2 passes through OY and is different from XOY and YOZ.

And so it follows that if we have determined ϕ_2 two things might happen:

(1) $\phi_2 = 0$ degenerates into two planes which are two perpendicular galactic meridians, and there will be then an indetermination of 90° in the longitude of the axis of rotation, as in Oort's method.

(2) $\phi_2 = 0$ degenerates into planes, one of them, P_1 , being a galactic meridian, and the other one being quite different from the galactic meridian perpendicular to P_1 . In this case the axis of rotation ωu is in the plane P_1 , and the indetermination of 90° alluded to before has vanished.

7. To determine ϕ_2 I have used the 2146 radial velocities of the Lick Catalogue, published in 1928. The stars in this catalogue are all nearer than 300 parsecs—that is to say, situated inside the local cloud.

Here is the result. The cone $\phi_2 = 0$ degenerates into two planes, one of these planes being nearly the galactic meridian of longitude 240° , the other not being far from the galactic plane.

According to this result the stars in the neighbourhood of the sun revolve around an axis situated in longitude 243° , the angular velocity Ω depends chiefly on the distance from the plane of symmetry of the cloud, and the sun is situated out of this plane.

8. I shall not quote figures because, desiring to strengthen this result, I have studied the question again, using all the radial velocities now known (about 5500) and grouping the stars in six categories according to their distances from the sun. The results I have obtained will soon be published. I will only say that the longitudes obtained by means of the first two groups for the axis of rotation are 239° and 242° . It appears, too, that f depends chiefly on z .

I shall communicate the numerical particulars when the work is finished.

IV. *Sir Frank Dyson's Investigation.*

9. Sir Frank Dyson has published recently (4) an investigation on the proper motions in longitude of the stars of types B and A in Boss's Catalogue. He has represented these motions by a series—

$$\mu_G = \alpha_0 + \alpha_1 \sin G + \beta_1 \cos G + \alpha_2 \sin 2G + \beta_2 \cos 2G + \alpha_3 \sin 3G + \beta_3 \cos 3G + \dots$$

and has calculated the coefficients α_i and β_i .

10. G_0 being the longitude of the axis of rotation, the terms in $2G$ and $3G$ are in the form

$$A \cos 2(G - G_0) + A' \cos 3(G - G_0) + \dots$$

Trying to determine G_0 from α_i, β_i one finds that (1) α_2, β_2 determine G_0 within $K \cdot 90^\circ$ * by means of

$$\tan 2G_0 = \frac{\alpha_2}{\beta_2}.$$

(2) α_3, β_3 determine G_0 within $K \cdot 60^\circ$ by means of

$$\tan 3G_0 = \frac{\alpha_3}{\beta_3}.$$

Now Sir Frank Dyson finds $G_0 = 330^\circ$ by means of α_2 and β_2 , and $G_0 = 355^\circ$ by means of α_3 and β_3 .

It results that the possible solutions are :

				G_0				
Terms in $2G$	60°	150°			240°	330°		
Terms in $3G$	55°	115°	175°	235°	295°		355°	

The two values 240° and 235° are nearer to one another than, for instance, 330° and 295° .

We conclude that the study of the proper motions of stars of types B and A leads us to admit 240° as the longitude of the axis of rotation instead of 330° .

* K is a positive or negative integer.

V. Study of the Apex.

11. Everyone has in mind the beautiful discovery of the asymmetrical drift made by Strömberg (5). If we draw from the origin of the co-ordinates vectors OV proportional to the velocity of the sun with regard to various groups of stars, the points v are in a straight line, parallel to the galactic plane, in the direction of longitude 240° .

This phenomenon has been rightly interpreted as expressing the diversity of the velocities of the various considered groups in their rotation around an axis at the longitude 330° .

Let us notice that most of the groups considered by Strömberg are outside the local cloud.

12. There is another asymmetrical drift which expresses the rotation of the stars around an axis at the longitude 240° .

In a note to the *Comptes Rendus* and in a paper which will be published in the *Bulletin de l'Observatoire de Lyon* I have studied the positions of the apex derived from the considerations of stars located within the local cloud.

These positions are arranged in a great circle of the celestial sphere. This great circle C cuts the galactic plane in a point at the longitude 330° —that is, at 90° from the centre of the local cloud. This phenomenon is specially plain as regards the stars in the neighbourhood of the North Pole.

13. It has often been pointed out that the declination of the apex differs considerably from one group of stars to another. The second asymmetrical drift we have just mentioned accounts for this fact, the great circle C being nearly an equatorial meridian. One must observe, however, that the R.A. of the apex increases at the same time with the declination; so that the asymmetrical drift cannot be explained, as has often been attempted, by admitting that the proper motions in declination are affected by an error of the form $A \cos \delta$, for such an error would not modify the R.A. of the apex.

VI. Some Considerations of Dynamics.

14. Once the existence of the local cloud is admitted, which the works of Charlier, Shapley, and Seares seem to have established, one is led to admit its permanence in time too.

Now as a consequence of the differential rotation of the galaxy, such a cluster would break up if not revolving on its axis.

Let us consider two stars moving in the gravitational field of the galaxy and being, at a given moment, under certain initial conditions exceedingly near in position and in velocity. The difference between their times of revolution around the centre of the galaxy will be a quantity ΔP very small as compared with the time P of their revolution. The orbits of these stars will be nearly the same. But after a number of revolutions of the order of $\frac{P}{\Delta P}$ the two stars will find themselves in two quite different points of their common orbit.

Let us consider now a cluster of stars. After a certain time the stars would be scattered over a ring which would outline the orbit of the cluster, unless the mutual attraction of the stars in this cluster held them together. But mutual attraction in a cluster implies acceleration directed toward the centre of the cluster. Therefore it is impossible even to conceive of the existence of the local cloud without admitting its rotation on its own axis, this being a consequence of the differential rotation of the galaxy itself.

15. The works of Oort and Plaskett concerning distant stars show a rotation which seems to occur around an axis situated at the longitude of 325° . It is not to be denied that the galaxy revolves around its centre situated in Sagittarius, and the local cloud as a whole must participate in this rotation.

The movement of the nearest stars would then be composed of two rotations—a rotation around the centre of the local cloud, to which would be added a rotation of this cloud around the centre of the galaxy.

This movement is not unlike the movement of the moon in the solar system.

Summary.

I have shown that, until now, the results obtained on the question of the galactic rotation can be given two interpretations—they may express the rotation of the stars around the centre of the galaxy (longitude 325°), or around the centre of the local cloud (longitude 243°).

The statistical study of the longitudes of the axis of rotation inclines my opinion to the second hypothesis.

A deeper study of the radial velocities favours clearly the rotation of the local cloud.

The results obtained by Sir Frank Dyson are easily accounted for by this hypothesis, and so is the distribution of the positions of the apex.

Lastly, we have important dynamical reasons for thinking that the local cloud of Charlier and Shapley revolves around its centre.

Without pretending to have settled the question concerning the position of the axis of rotation in the galaxy, without even pretending to be in the right, I have thought it my duty to bring forward these considerations to astronomers, as I should be only too happy if the criticisms my work is liable to raise could help us to a better knowledge of stellar movements.

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Spectroscopic Parallaxes of B-type Stars (Sixth Paper): The Spectral Types and Parallaxes of 175 Stars. By D. L. Edwards, A.R.C.S., D.I.C.

(Communicated by the Director of the Norman Lockyer Observatory.)

The methods used in determining spectroscopic parallaxes of B-type stars at the Norman Lockyer Observatory have already been sufficiently described in previous papers of this series,* and nothing further need be said here on that point. The present paper contains a list of the spectral classifications and parallaxes of 175 stars based on photographs taken since the publication of the last list.† All the photographs were taken at Sidmouth with the 12-inch prismatic camera, which has a prism of 20° angle giving a dispersion such that the distance between H β and H δ is 21.4 mm. The plates used were chiefly Ilford "Iso-Zenith" (speed 700 H. & D.), and in some cases Imperial "Eclipse Soft" (speed 850). It has previously been mentioned that the effect of different emulsions on the character of the spectra obtained is practically negligible from the point of view of parallax work, and this conclusion has been justified by taking spectra of the same stars on the above two brands of plates, which gave identical results under the wedge. The focal curve of the objective is sufficiently flat over the region investigated (chiefly from λ 4481 to λ 3933) to give good definition without the necessity of altering the settings for different lines. The spectral types were determined from measurements of the intensities of the principal lines of He, H, C, O, Si, Mg, and Ca, as already explained.

* *Monthly Notices*, **84**, 366, 1924; **87**, 364, 1927.

† *Ibid.*, **88**, 175, 1928.

The classification into line-character groups was made by comparison with sets of standard negatives for each spectral subdivision.

In Table I. are given the final results for 175 stars. The number of the star in the Henry Draper Catalogue is given in column 1. In column 2, when the star has no special designation, the Boss number is given if available, otherwise the H.R. or B.D. number. Columns 3 and 4 give the position for 1900; 5 and 6 the apparent magnitude and spectral type from the Henry Draper Catalogue; 7 and 8 the spectral classification as determined at the Norman Lockyer Observatory; and the last two columns the deduced absolute magnitude and parallax. The parallaxes in column 10 have been corrected, where necessary, on account of the spectroscopic binary correction as suggested in a recent note on this source of error.* The parallaxes of non-binary stars were multiplied by 1.05, those of spectroscopic binaries showing one spectrum by 0.92, and those showing two spectra by 0.84.

In Table II. are given the spectral classifications, absolute magnitudes, and parallaxes as determined elsewhere for the purpose of comparison with the present results. They consist chiefly of spectroscopic and moving cluster parallaxes, with the addition of two dynamical values and one trigonometrical. Columns 2 and 3 give the sources from which the parallaxes were obtained, together with the authors for which the following abbreviations are used:—AJ = Adams and Joy; † LS = Lindblad and Schalén; ‡ R = Rasmuson; § K = Kapteyn; || JF = Jackson and Furner.¶ Columns 4, 5, and 6 give the spectra, absolute magnitudes, and parallaxes as published by these authors; and column 7 the parallaxes from Table I. for comparison.

TABLE I.
Classifications and Parallaxes of 175 Stars.

1.	2.	3.		4.	5.	6.	7.	8.	9.	10.
H.D.	Name.	1900.			m.	Spec. (H.D.)	N.L.O.		M.	π .
		R.A.	Dec.							
		^h o	^m s	^s °						
584	HR 28	5.2	+56	36	6.5	B8	B8	<i>ns</i>	-0.1	0.005
1064	Boss 34	9.8	-10	7	5.8	B9	B9	<i>n</i>	+0.3	0.008
1438	26 And.	13.4	+43	14	6.0	B9	B8	<i>ns</i>	-0.1	0.006
2054	Boss 71	19.7	+52	30	5.7	B9	B8	<i>s</i>	-0.5	0.006
3038	16 Cas.	28.6	+66	12	6.4	B9	B8	<i>n</i>	+0.2	0.005
3322	HR 149	31.1	+26	42	6.3	B8	B9	<i>ns</i>	0.0	0.006
3924	HR 181	36.7	+58	13	6.1	B9	B9	<i>ns</i>	0.0	0.006
5550	Boss 208	52.2	+65	49	6.0	B9	A0	<i>s</i>	-0.1	0.006
6676	Boss 247	I 2.4	+57	44	5.7	B8	B7	<i>ns</i>	-0.2	0.007
6960	Boss 263	5.0	+63	40	5.5	B9	B9	<i>ns</i>	0.0	0.008

* *Monthly Notices*, 88, 695, 1928. † *Ap. J.*, 57, 294, 1928.

‡ *Upsala Meddelanden*, No. 17, 1927.

§ *Lund Meddelanden*, II, No. 26, 1921.

|| *Ap. J.*, 47, 255, 1918.

¶ *Monthly Notices*, 81, 2, 1920; 83, 4, 1922.

TABLE I.—*continued.*

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
H.D.	Name.	1900.		m.	Spec. (H.D.)	N.L.O.		M.	π .
		R.A.	Dec.						
		^h ^m	^o [']						
7019	45 And.	1 5.5	+37 12	5.8	B8	B7	s	-0.6	0.006
7157	HR 354	6.8	+61 10	6.3	B9	B9	ns	0.0	0.006
9531	Boss 340	28.5	+36 44	5.8	B9	B9	ns	0.0	0.007
9766	101 Psc.	30.4	+14 9	6.2	B9	B9	ns	0.0	0.006
10260	HR 481	35.3	+60 32	6.6	B8	B9	s	-0.3	0.004
10362	HR 488	36.1	+60 55	6.5	B8	B7	s	-0.6	0.004
10425	44 Cas.	36.6	+60 3	5.8	B9	B9	ns	0.0	0.006
11291	2 Per.	45.8	+50 18	5.6	B9	B9	ns	0.0	0.008
11857	HR 561	51.4	+61 12	6.1	B8	B7	ns	-0.2	0.006
14818	10 Per.	2 18.2	+56 10	6.2	B2	B2	ss	-2.7	0.002
14951	ξ Ari.	19.5	+10 9	5.5	B5	B6	n	-0.3	0.006
16004	HR 746	29.2	+39 15	6.3	B9	B9	ns	0.0	0.006
16219	HR 760	31.1	+39 27	6.4	B8	B6	ns	-0.5	0.004
18633	5 Eri.	54.7	- 2 52	5.5	B9	B9	n	+0.3	0.009
19065	Boss 699	58.9	+63 41	5.8	B9	B9	s	-0.3	0.006
20315	30 Per.	3 11.0	+43 39	5.4	B5	B7	ns	-0.2	0.007
21203	HR 1033	20.2	+59 54	6.5	B8	B9	ns	0.0	0.005
22316	HR 1094	30.5	+56 37	6.3	B9	B9	s	-0.3	0.005
22780	Boss 833	34.6	+37 16	5.6	B5	B6	n	-0.3	0.007
23383	HR 1147	39.7	+55 37	6.0	B9	B9	n	+0.3	0.008
24131	Boss 886	45.5	+34 3	5.7	B3	B2	ss	-2.7	0.002
25340	35 Eri.	56.5	- 1 50	5.3	B5	B5	ns	-0.7	0.007
32343	11 Cam.	4 57.4	+58 50	5.3	B3p	B3p	nn	-0.8	0.006
33949	κ Lep.	5 8.7	-13 3	4.5	B8	B7	ns	-0.2	0.013
34233	15 Cam.	10.8	+58 1	6.2	B3	B4	ns	-1.0	0.003
34759	ρ Aur.	14.7	+41 43	5.1	B3	B3	n	-1.1	0.006
35600	Boss 1310	20.8	+30 7	5.7	B9	B9	ss	-1.0	0.005
36653	35 Ori.	28.2	+14 15	5.6	B3	B4	ns	-1.0	0.005
38831	30 Cam.	43.5	+58 55	6.1	B9	B9	ns	0.0	0.006
39317	137 Tau.	46.7	+14 9	5.6	B9	B9	ns	0.0	0.007
41117	χ^2 Ori.	58.0	+20 8	4.7	B2p	B2	s	-2.3	0.004
43153	72 Ori.	6 9.6	+16 11	5.3	B9	B8	s	-0.3	0.008
47100	ψ^3 Aur.	31.9	+39 59	5.3	B8	B7	ns	-0.2	0.008
47395	54 Aur.	33.3	+28 21	5.8	B8	B8	ns	-0.1	0.007
47964	Boss 1710	36.0	+ 0 36	5.6	B9	B8	s	-0.5	0.006
49606	33 Gem.	44.1	+16 19	5.7	B8	B8	s	-0.5	0.009
52860	HR 2645	57.7	+47 54	6.4	B9	B9	ns	0.0	0.006

TABLE I.—continued.

1.	2.	3.		4.	5.	6.	7.	8.	9.	10.
H.D.	Name.	1900.			m.	Spec. (H.D.)	N.L.O.	M.	π.	
		R.A.	Dec.							
		h	m	°						
53744	HR 2669	7	1.1	+28 21	6.2	B ₉	B ₉ <i>ns</i>	0.0	0.006	
53929	HR 2676		1.8	+ 5 4	6.0	B ₈	B ₈ <i>ns</i>	-0.1	0.006	
56386	HR 2757		11.7	+31 9	6.0	B ₉	B ₉ <i>ns</i>	0.0	0.007	
57608	HR 2801		16.9	+ 0 22	6.0	B ₈	B ₉ <i>s</i>	-0.3	0.006	
57742	HR 2809		17.5	+66 32	6.3	B ₉	B ₉ <i>n</i>	+0.3	0.007	
58050	HR 2817		18.8	+15 43	6.4	B ₃	B ₃ <i>s</i>	-1.8	0.002	
58729	+15° 1574		21.8	+15 30	7.0	B ₈	B ₉ <i>ns</i>	0.0	0.004	
66684	HR 3164		59.6	+27 49	6.2	B ₉	B ₉ <i>ns</i>	0.0	0.006	
68099	HR 3201	8	5.9	+10 7	6.1	B ₉	B ₉ <i>s</i>	-0.3	0.006	
68194	+48° 1621		6.3	+48 35	6.8	B ₉	A ₀ <i>ns</i>	+0.3	0.005	
71554	+54° 1231		23.0	+54 27	6.7	B ₉	B ₉ <i>ns</i>	0.0	0.005	
74604	Boss 2344		39.8	+67 5	6.2	B ₈	B ₇ <i>ns</i>	-0.2	0.006	
75894	+53° 1291		47.7	+53 11	7.4	B ₉	B ₉ <i>n</i>	+0.3	0.004	
77770	+50° 1608		59.5	+50 1	7.5	B ₅	B ₃ <i>ss</i>	-2.3	0.001	
83953	I Hydræ.	9	36.7	-23 8	4.7	B _{2p}	B _{3p} <i>nn</i>	-0.8	0.008	
91130	33 LMi.	10	26.2	+32 54	5.8	B ₉	A ₀ <i>ns</i>	+0.3	0.008	
93427	HR 4215		42.1	+65 40	6.2	B ₉	A ₀ <i>ns</i>	+0.3	0.007	
99966	+49° 2061	11	25.0	+49 29	7.1	B ₉	B ₉ <i>n</i>	+0.3	0.005	
105262	+13° 2491	12	2.1	+13 33	7.0	B ₉	B ₉ <i>s</i>	-0.3	0.004	
112504	-8° 3456		52.1	- 8 22	6.8	B ₉	A ₀ <i>n</i>	+0.6	0.006	
114376	15 CVn.	13	5.1	+39 4	6.2	B ₉	B ₆ <i>s</i>	-0.8	0.004	
115301	+22° 2564		11.2	+21 55	7.2	B ₉	B ₉ <i>n</i>	+0.3	0.004	
115612	HR 5018		13.2	+68 56	6.1	B ₉	B ₉ <i>ns</i>	0.0	0.006	
124683	Boss 3653	14	9.9	-17 44	5.6	B ₉	A ₀ <i>ns</i>	+0.3	0.009	
127304	HR 5422		25.5	+32 14	6.0	B ₉	A ₀ <i>s</i>	-0.1	0.006	
129433	Boss 3756		37.4	-24 34	5.8	B ₉	B ₉ <i>n</i>	+0.3	0.008	
129956	108 Vir.		40.4	+ 1 9	5.5	B ₉	A ₀ <i>ns</i>	+0.3	0.009	
130870	+25° 2841		45.5	+25 9	6.6	B ₉	A ₀ <i>n</i>	+0.6	0.007	
137389	HR 5731	15	20.7	+62 23	5.8	B ₉	A ₀ <i>s</i>	-0.1	0.007	
138527	τ ² Ser.		27.6	+16 24	6.1	B ₉	B ₉ <i>n</i>	+0.3	0.007	
142763	HR 5931		51.2	+18 55	6.2	B ₉	B ₆ <i>s</i>	-0.8	0.004	
143187	HR 5949		53.9	+59 12	6.2	B ₉	B ₉ <i>ns</i>	0.0	0.006	
147550	HR 6096	16	17.5	- 1 51	6.1	B ₉	A ₀ <i>ns</i>	+0.3	0.007	
149881	+14° 3086		32.4	+14 40	6.6	B ₂	B ₂ <i>s</i>	-2.3	0.002	
151133	16 Oph.		40.4	+ 1 12	6.0	B ₉	B ₈ <i>ns</i>	-0.1	0.005	
156377	+18° 3336	17	12.1	+18 8	6.9	B ₉	B ₉ <i>ns</i>	0.0	0.004	
156651	+31° 2993		13.7	+31 37	7.2	B ₅	B ₉ <i>s</i>	-0.3	0.003	

TABLE I.—*continued.*

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
H.D.	Name.	1900.			Spec. (H.D.)	N.L.O.		M.	π.
		R.A.	Dec.						
157741	HR 6482	17 ^h 20 ^m 0	+15° 43'	6.3	B9	B9	<i>n</i>	+0.3	0.007
158261	HR 6506	23.2	+34 47	5.9	B9	A2	<i>ns</i>	+1.0	0.011
159082	HR 6532	27.6	+12 0	6.2	B9	B9	<i>ns</i>	0.0	0.005
159358	Boss 4448	29.2	-11 10	5.6	B8	B8	<i>n</i>	+0.2	0.009
161695	HR 6619	41.9	+31 33	6.3	B9	A2	<i>s</i>	+0.5	0.007
163966	+45° 2626	53.8	+45 1	6.8	B9	B9	<i>ns</i>	0.0	0.005
164432	HR 6719	56.0	+ 6 16	6.2	B3	B3	<i>s</i>	-1.8	0.003
164447	HR 6720	56.1	+19 31	6.4	B9	B9	<i>ns</i>	0.0	0.006
165402	Boss 4573	18 0.7	- 8 20	5.8	B8	B8	<i>ns</i>	-0.1	0.006
168199	HR 6851	13.5	+13 44	6.2	B5	B4	<i>n</i>	-0.7	0.004
168797	HR 6873	16.5	+ 5 24	6.0	B5	B4	<i>ns</i>	-1.0	0.004
170200	Boss 4673	23.1	+ 6 8	5.6	B8	B8	<i>s</i>	-0.5	0.006
170973	HR 6958	27.1	+ 3 55	6.3	B9	A2	<i>s</i>	+0.5	0.007
171247	HR 6967	28.6	+ 8 12	6.3	B8	B8	<i>s</i>	-0.5	0.005
171406	HR 6971	29.6	+30 49	6.4	B3	B4	<i>n</i>	-0.7	0.004
173087	HR 7033	38.5	+34 38	6.1	B5	B4	<i>ns</i>	-1.0	0.004
173370	4 Aql.	39.8	+ 1 57	5.0	B5	B8	<i>n</i>	+0.2	0.012
174853	HR 7109	47.5	+13 51	6.1	B9	B9	<i>ns</i>	0.0	0.006
177109	HR 7212	58.2	+33 29	6.2	B3	B4	<i>n</i>	-0.7	0.004
178329	Boss 4870	19 3.1	+41 16	6.2	B3	B3	<i>ns</i>	-1.4	0.003
181440	27 Aql.	15.4	- 1 4	5.5	B9	B9	<i>s</i>	-0.3	0.006
182308	HR 7361	19.0	+64 13	6.3	B9	B9	<i>s</i>	-0.3	0.005
182691	Boss 4957	20.8	+50 5	6.3	B9	B8	<i>ns</i>	-0.1	0.006
183227	HR 7397	23.4	+ 2 43	5.9	B9	B8	<i>s</i>	-0.5	0.006
184961	HR 7452	31.8	+22 22	6.1	B9	B9	<i>n</i>	+0.3	0.007
185037	11 Cyg.	32.2	+36 43	5.9	B9	B9	<i>n</i>	+0.3	0.008
185330	HR 7467	33.5	+38 10	6.4	B8	B5	<i>s</i>	-1.0	0.003
187235	Boss 5054	43.9	+38 9	5.7	B9	B9	<i>n</i>	+0.3	0.009
188439	HR 7600	50.1	+47 34	6.2	B2	B2	<i>n</i>	-1.6	0.003
189066	HR 7620	53.1	+35 59	6.0	B3	B4	<i>ns</i>	-1.0	0.004
191243	HR 7699	20 3.8	+34 8	6.1	B8	B6	<i>ss</i>	-1.4	0.003
191877	HR 7716	7.0	+21 35	6.1	B0	B0	<i>ns</i>	-3.4	0.001
192044	20 Vul.	7.8	+26 11	5.9	B8	B9	<i>n</i>	+0.3	0.007
192987	HR 7757	12.8	+36 45	6.3	B5	B5	<i>n</i>	-0.5	0.004
193536	HR 7777	15.6	+46 0	6.3	B1	B3	<i>ns</i>	-1.4	0.003
193722	HR 7786	16.7	+46 31	6.2	B9	A0	<i>ns</i>	+0.3	0.007
193964	71 Dra.	18.0	+61 56	5.6	B9	B9	<i>ns</i>	0.0	0.008

TABLE I.—continued.

1. H.D.	2. Name.	3.		4.	5.	6.	7.	8.	9.	10.
		1900.				Spec. (H.D.)				
		R.A. h m		Dec. ° '	m.		N.L.O.		M.	π .
194335	Boss 5236	20 20.0		+37 10	5.7	B ₃	B ₃	<i>nn</i>	-0.8	0.004
195554	Boss 5264	27.0		+55 44	5.9	B ₉	B ₉	<i>ns</i>	0.0	.007
196178	Boss 5283	30.6		+46 21	5.6	B ₉	B ₉	<i>s</i>	-0.3	.007
196504	27 Vul.	32.8		+26 7	5.5	B ₉	B ₉	<i>n</i>	+0.3	.008
196712	Boss 5300	34.0		- 2 46	6.3	B ₉	B ₇	<i>n</i>	0.0	.006
197018	Boss 5313	35.9		+40 13	5.9	B ₈	B ₉	<i>s</i>	-0.3	.006
199661	Boss 5394	53.6		+56 29	6.1	B ₃	B ₃	<i>ns</i>	-1.4	.003
199892	HR 8036	54.9		+41 33	6.0	B ₉	B ₇	<i>n</i>	0.0	.006
199955	Boss 5405	55.3		+50 4	5.5	B ₈	B ₈	<i>s</i>	-0.5	.007
200340	HR 8054	57.8		- 1 19	6.3	B ₈	B ₈	<i>s</i>	-0.5	.005
200575	+56° 2523	59.2		+56 40	6.7	B ₈	B ₄	<i>n</i>	-0.7	.003
200614	Boss 5421	59.4		+56 16	5.7	B ₉	B ₈	<i>ns</i>	-0.1	.007
201819	HR 8105	21 7.0		+35 53	6.4	B ₁	B ₁	<i>n</i>	-2.3	.002
201834	Boss 5447	7.1		+53 9	5.7	B ₉	B ₉	<i>ss</i>	-1.0	.004
201888	HR 8109	7.4		+62 53	6.5	B ₈	B ₆	<i>ns</i>	-0.5	.004
202214	Boss 5453	9.3		+59 35	5.7	B ₂	B ₁	<i>s</i>	-2.9	.002
202862	HR 8144	13.6		+42 16	6.1	B ₈	B ₆	<i>n</i>	-0.3	.006
203025	HR 8153	14.6		+58 10	6.4	B ₃	B ₃	<i>ns</i>	-1.4	.003
203245	Boss 5478	16.0		+49 6	5.7	B ₅	B ₆	<i>ss</i>	-1.4	.003
205551	Boss 5548	31.0		+51 15	6.0	B ₉	B ₈	<i>ns</i>	-0.1	.006
206540	HR 8292	37.6		+10 21	6.0	B ₈	B ₇	<i>s</i>	-0.6	.005
207516	HR 8338	44.4		+38 11	5.8	B ₉	B ₈	<i>ns</i>	-0.1	.007
207840	Boss 5621	46.9		+19 22	5.7	B ₉	B ₇	<i>ss</i>	-1.3	.004
207857	HR 8349	47.0		+39 5	6.2	B ₉	B ₉	<i>s</i>	-0.3	.005
208095	HR 8357	48.8		+55 20	5.5	B ₃	B ₆	<i>nn</i>	-0.1	.008
208501	13 Cep.	51.5		+56 8	6.0	B _{9p}	B ₆	<i>ss</i>	-1.4	.003
208682	Boss 5642	52.9		+64 52	5.9	B ₂	B ₃	<i>ns</i>	-1.4	.004
208947	HR 8384	54.7		+65 41	6.3	B ₃	B ₃	<i>s</i>	-1.8	.003
209288	HR 8397	57.1		+10 29	6.4	B ₈	B ₄	<i>ns</i>	-1.0	.003
209339	HR 8399	57.6		+62 0	6.5	B ₀	B ₁	<i>s</i>	-2.9	.001
209419	Boss 5664	58.2		+52 24	5.7	B ₅	B ₅	<i>ns</i>	-0.7	.006
209961	HR 8427	22 2.0		+47 45	6.2	B ₃	B ₃	<i>ns</i>	-1.4	.003
212043	+56° 2755	16.4		+56 25	6.5	B ₈	B ₇	<i>ns</i>	-0.2	.005
212454	HR 8535	19.3		+56 47	6.2	B ₈	B ₈	<i>ns</i>	-0.1	.006
212883	HR 8549	22.4		+36 57	6.4	B ₃	B ₃	<i>s</i>	-1.8	.002
212978	HR 8553	23.1		+39 19	6.1	B ₃	B ₃	<i>s</i>	-1.8	.003
214240	Boss 5834	31.8		+49 33	6.2	B ₃	B ₃	<i>s</i>	-1.8	.003

TABLE I.—*continued.*

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
H.D.	Name.	1900.			Spec. (H.D.)	N.L.O.		M.	π.
		R.A.	Dec.						
		h m	o ' "						
216057	HR 8682	22 44.6	+53 53	6.1	B8	B ₄	<i>nn</i>	-0.5	0.005
216523	HR 8705	48.5	+49 53	6.4	B ₉	B8	<i>s</i>	-0.5	0.004
216538	HR 8706	48.6	+39 37	6.2	B8	B ₇	<i>s</i>	-0.6	0.005
216831	Boss 5912	51.1	+35 50	5.6	B ₉	B ₇	<i>ns</i>	-0.2	0.007
217062	+59° 2607	52.8	+59 25	7.1	B ₉	B ₆	<i>ns</i>	-0.5	0.003
217348	HR 8745	55.0	+59 16	6.4	B ₉	B8	<i>s</i>	-0.5	0.004
217543	HR 8758	56.3	+38 10	6.4	B ₃	B ₃	<i>s</i>	-1.8	0.004
217811	HR 8768	58.2	+43 31	6.3	B ₃	B ₃	<i>ss</i>	-2.3	0.002
217817	+59° 2629	58.3	+59 18	6.9	B ₃	B ₃	<i>ns</i>	-1.4	0.002
217943	HR 8777	59.2	+59 54	6.6	B ₅	B ₅	<i>ns</i>	-0.7	0.004
218537	HR 8808	23 3.7	+63 6	6.2	B ₃	B ₃	<i>ns</i>	-1.4	0.003
219634	HR 8854	12.2	+61 25	6.5	B8	B ₃	<i>ns</i>	-1.4	0.003
219749	HR 8861	13.2	+44 57	6.3	B _{9p}	B ₉	<i>n</i>	+0.3	0.007
219927	HR 8873	14.6	+34 15	6.1	B ₉	B ₇	<i>s</i>	-0.6	0.005
224103	26 Psc.	50.0	+ 6 31	6.1	B ₉	B ₉	<i>ns</i>	0.0	0.006
224404	HR 9063	52.5	+59 28	6.4	B8	B8	<i>ns</i>	-0.1	0.005
224906	Boss 6168	56.6	+41 49	6.1	B ₉	B ₉	<i>s</i>	-0.3	0.006

TABLE II.

Stars in Table I. for which other Parallaxes have been published.

1. H.D.	2. Source.	3. Auth.	4. Spectrum.	5. <i>M.</i>	6. <i>π.</i>	7. <i>π_L.</i>
1438	Spec.	AJ	B9 <i>s</i>	-0.2	0.006	0.006
	"	LS	A0 <i>σ</i>	0.0	0.006	
3038	"	LS	A0 <i>σ</i>	0.0	0.005	0.005
5550	"	LS	A0 <i>μ</i>	+0.7	0.009	0.006
6676	"	LS	B8 <i>σ</i>	0.0	0.007	0.007
6960	"	AJ	B9 <i>s</i>	-0.2	0.007	0.008
	"	LS	B8 <i>μ</i>	+0.7	0.011	
	Cluster	R			0.010	
7019	Spec.	LS	A0 <i>σ</i> +	-0.7	0.005	0.006
9531	"	AJ	B9 <i>n</i>	+0.5	0.009	0.007
	Trig.	MW			0.008	
9766	Spec.	AJ	B9 <i>n</i>	+0.5	0.007	0.006
10425	"	LS	A0 <i>σ</i> +	-0.7	0.005	0.006
14818	"	AJ	B1 <i>n</i>	-2.8	0.002	0.002
14951	"	AJ	B8 <i>n</i>	+0.2	0.009	0.006

TABLE II.—*continued.*

1. H.D.	2. Source.	3. Auth.	4. Spectrum.	5. M.	6. π .	7. π_E .
18633	Spec.	AJ	B ₉ <i>n</i>	+0.5	0.010	0.009
19065	"	AJ	B ₉ <i>s</i>	-0.2	.006	.006
20315	"	AJ	B ₇ <i>n</i>	0.0	.008	.007
	"	LS	B σ	0.0	.008	
	Cluster	R			.009	
22780	Spec.	AJ	B ₈ <i>n</i>	0.0	.008	.007
	Cluster	R			.008	
24131	Spec.	AJ	B ₃ <i>n</i>	-0.9	.005	.002
25340	"	AJ	B ₄ <i>n</i>	-0.8	.006	.007
	Cluster	K			.009	
32343	Spec.	LS	B τ -	-1.8	.004	.006
33949	Cluster	K			.006	.013
34233	"	R			.008	.003
34759	Spec.	AJ	B ₅ <i>n</i>	-0.4	.008	.006
	"	LS	B ₈ σ +	-0.7	.007	
	Cluster	R			.011	
35600	Spec.	AJ	B ₉ <i>s</i>	0.0	.007	.005
	"	LS	B-A ₀ τ -	-1.8	.003	
36653	Spec.	AJ	B ₄ <i>n</i>	-0.6	.006	.005
	Cluster	K			.006	
39317	"	K			.007	.007
41117	Spec.	AJ	B ₂ <i>s</i>	-2.3	.004	.004
	"	LS	B τ	-3.4	.002	
	Cluster	K			.007	
43153	Spec.	AJ	B ₈ <i>n</i>	+0.3	.010	.008
	Cluster	K			.009	
49606	Spec.	AJ	B ₈ <i>s</i>	-0.4	.006	.006
74604	"	LS	A ₀ σ -	0.0	.006	.006
91130	"	AJ	B ₉ <i>n</i>	+0.5	.009	.008
	"	LS	A ₀ μ	+0.7	.010	
114376	"	LS	B ₈ σ +	-0.7	.004	.004
124683	"	AJ	B ₉ <i>s</i>	-0.4	.006	.009
129433	"	AJ	A ₀ <i>n</i>	+0.9	.010	.008
138527	"	AJ	B ₉ <i>n</i>	+0.5	.008	.007
165402	"	AJ	B ₆ <i>n</i>	-0.2	.006	.006
	Cluster	K			.005	
170200	Spec.	Ab.	B ₈ <i>n</i>	+0.3	.009	.006
173370	"	AJ	B ₈ <i>n</i>	+0.2	.011	.012

TABLE II.—*continued.*

1. H.D.	2. Source.	3. Auth.	4. Spectrum.	5. M.	6. π .	7. π_E .
178329	Spec.	LS	A0 $\sigma+$	-0.7	0.004	0.003
187235	"	LS	A0 $\sigma-$	0.0	0.007	0.009
194335	"	LS	B-A0 τ	-3.4	0.002	0.004
197018	Dynam.	JF			0.006	0.006
199955	Spec.	AJ	B8 n	+0.1	0.008	0.007
202214	Dynam.	JF			0.005	0.002
208095	Spec.	AJ	B6 n	-0.2	0.007	0.008
	"	LS	A0 $\sigma-$	0.0	0.008	
208501	"	LS	B τ	-3.4	0.001	0.003
208682	"	LS	B τ	-3.4	0.001	0.004
209419	"	LS	B8 $\sigma+$	-0.7	0.005	0.006
214240	"	LS	B $\tau-$	-1.8	0.003	0.003
216831	"	LS	A0 σ	0.0	0.008	0.007
224906	"	LS	B8 $\sigma+$	-0.7	0.004	0.006

A summary of these results is given in Table III., in which the mean values of all the parallaxes from the different sources individually and collectively are shown, together with the corresponding means from the parallaxes of Table I. It will be seen from this table that the parallaxes of the present list are slightly smaller in the average than those of the other authors quoted, though this difference is almost negligible in the case of the spectroscopic parallaxes and is reversed in the case of Lindblad's and Schalén's results. In any case the difference is very small, the maximum value being 0".0011 (for cluster parallaxes) whilst for the mean of all parallaxes it is only 0".0005. The largest individual difference is 0".007, in the case of κ Leporis (H.D. 33949); but as this is marked as very doubtful in Kapteyn's list the error probably lies in the cluster parallax.

TABLE III.

	No. of Parallaxes.	Mean π .	Mean π_E .	$\bar{\pi}-\bar{\pi}_E$.
Adams and Joy	26	0.0073	0.0066	+0.0007
Lindblad and Schalén	25	0.0054	0.0057	-0.0003
Spectroscopic (all values)	52	0.0064	0.0062	+0.0002
Cluster parallaxes	12	0.0079	0.0068	+0.0011
All parallaxes	67	0.0067	0.0062	+0.0005

A comparison of the line-character groups given in Table I., column 8, with the corresponding groups as determined by Adams and Joy and by Lindblad and Schalén, is of some interest in view of the different

types of instrument used at the three observatories. This was done previously in *Monthly Notices*, 87, 383, 1927, with the material then available, but many more classifications have been published since that time both from Sidmouth and Upsala. In addition, the Upsala classification has been considerably revised, and with the more complete data now available the correspondence (or otherwise) of the different systems is better shown. Table IV. shows the frequencies of the various groups used at Mount Wilson and Upsala occurring in the five groups of the Sidmouth classification.

TABLE IV.

Frequency Distribution of Line-character Groups.

N.L.O.	Adams and Joy.		Lindblad and Schalén.						
	s.	n.	τ .	$\tau-$.	$\sigma+$.	σ .	$\sigma-$.	ρ .	μ .
ss	10	5	4	6	1	1	0	0	0
s	26	13	4	7	9	3	1	0	2
ns	18	50	1	11	17	10	3	0	2
n	2	34	1	8	6	2	0	0	1
nn	0	3	1	5	0	0	0	0	0

The definite correlation with Adams and Joy's groups is well shown in this table, and also the fact that the *ns* group is more closely related to the *n* group of Adams and Joy than to their *s* group. In the case of Lindblad and Schalén's classification the connection seems to be less obvious, though there is evidence of some sort of correlation with the first three groups, *ss* to *ns*. This evidence is more definite than was previously found to be the case, but it is rather surprising that so many τ stars are to be found amongst the *n* and *nn* groups, and that the winged types $\sigma-$, ρ , and μ , which might be expected to be numerous, are almost completely absent from these groups.

During the progress of this work some valuable researches by Dr. O. Struve have appeared which give considerable support both to the method employed in determining spectroscopic parallaxes of B-type stars and to the actual results obtained. In a research on the Stark effect in stellar spectra * he shows that the broadening of lines in early type spectra is largely due to this effect, and deduces a relationship between luminosity and line width which is in agreement with the corresponding relations used for spectroscopic parallaxes. Other broadening mechanisms presumably occur as well, thereby causing the spectroscopic absolute magnitudes to show a smaller dispersion than occurs in reality, but he concludes "that the Stark effect gives a satisfactory explanation for the observed difference of luminosity between 's' and 'n' stars." In addition, his work on detached calcium lines,† in which a relation is discovered between the intensities of these lines

* *Ap. J.*, 69, 173, 1929.† *Monthly Notices*, 89, 567, 1929.

and stellar distances, provides a valuable check on spectroscopic parallaxes which is independent of the standard data used in deducing the latter. The quantities deduced directly from the calcium line intensities are stellar distances and not absolute magnitudes, as in the spectroscopic method. Also, the results are not affected by any restriction on the dispersion in absolute magnitudes. It is therefore interesting to note that Struve finds a good correlation between his results and those found by spectroscopic methods at Mount Wilson and Sidmouth. The relation between line width and absolute magnitude also receives confirmation from these results, independently of the theoretical considerations mentioned above. The following summary gives the mean absolute magnitudes, as found by Struve, for stars of the various line-groups *ss* to *nn* which occur in his list:—

	<i>ss.</i>	<i>s.</i>	<i>ns.</i>	<i>n.</i>	<i>nn.</i>
Bo to B ₂	-6.1	-4.9	-4.3	-3.3	
B ₃	-5.3	-4.7	-3.8	-2.3	-1.2

In this summary all stars for which $D \geq 100$ units are omitted owing to the greater uncertainty of the calcium method for the nearer stars. The absolute magnitudes are all brighter than those used for spectroscopic parallaxes (owing partly to the much greater range covered by the calcium method and partly to the exclusion of the nearer stars), but the definite decrease in luminosity with increase in line width is well shown. Also, the existence of a pronounced relation of the same kind for B₃ stars alone shows that it is not a mere correlation of *M* and spectral type.

With regard to the use of line intensities for obtaining absolute magnitudes (which is one of the methods used at this observatory) the inclusion of the intensities of three lines in Struve's list gives a useful rough check on the method. These three lines ($H\gamma$, λ 4472, and λ 4388) are amongst those which have been used for parallax work, and although Struve finds no marked correlation between absolute magnitude and individual line intensities yet there appears to be some evidence of a relation if the differences of intensity of the Hydrogen and Helium lines are investigated. Grouping the stars according to Struve's absolute magnitudes and taking the means of the intensity differences $H\gamma - 4472$, $H\gamma - 4388$ from the same source, gives the following results:—

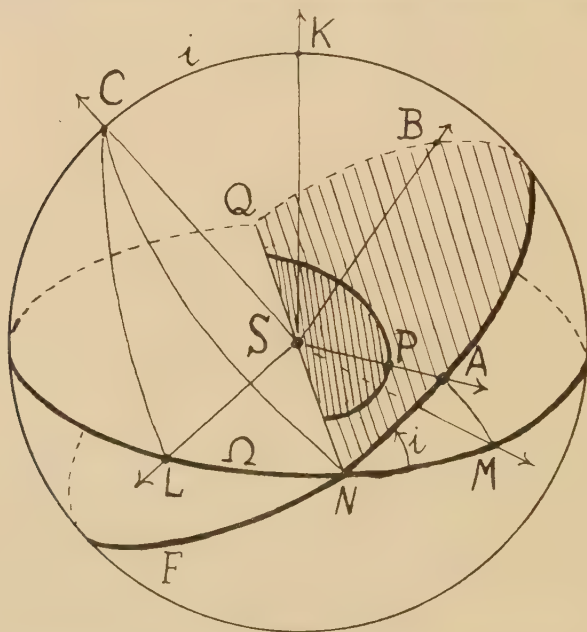
<i>M.</i>	-8.2.	-5.9.	-4.3.	-2.6.	-1.3.
$H\gamma - 4472$	1.7	3.7	3.9	4.8	6.7
$H\gamma - 4388$	5.4	5.8	6.0	7.0	8.8

As before, stars of distance 100 units or nearer have been omitted. These figures seem to lend support to the use of the above two intensity differences for spectroscopic parallax work, although, as might be expected, there is considerable dispersion in absolute magnitude for any specified intensity difference.

On the Derivation of the Elements of a Visual Binary Orbit by Kowalsky's Method. By W. M. Smart, M.A., D.Sc.

(Communicated by the Director of Cambridge Observatory.)

1. A detailed description of the well-known method, originally proposed by Kowalsky, of deriving the elements of a visual binary orbit is given by C. André (*Traité d'Astronomie Stellaire*, T. II.) and by R. G. Aitken (*The Binary Stars*). The mathematical analysis of the original



method is somewhat unwieldy and the following procedure is offered as an alternative.

2. The figure shows a sphere the centre of which is the primary star S. The great circle LNM defines the plane of the apparent orbit, which is at right angles to the line of sight. The great circle FNAB defines the plane of the true orbit of the companion relative to the primary. QSN is the line of nodes. Let SL define the direction, in the plane of the apparent orbit, of position angle 0° ; we shall suppose that position angles are measured from L towards N. Then LN is the longitude Ω of the node (the node chosen is such that Ω lies between 0° and 180°). Let the companion be nearest the primary at P; this point is periastron, and if A is the corresponding point on the sphere, the arc NA is denoted by ω . The angle ANM is the inclination i which we shall take for our purpose to lie between 0° and 90° (in general, the

sign of $\cos i$ is indeterminate). Let a and e denote as usual the semi-major axis (in seconds of arc) and the eccentricity of the companion's true orbit relative to the primary S. The elements whose derivation concerns us here are a , e , i , Ω , and ω .

3. The observations furnish the apparent orbit which is the orthogonal projection of the true ellipse on the plane LNM. Take rectangular axes SL, SM, and SK. Then the equation of the apparent ellipse is of the form

$$Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + 1 = 0. \quad (1)$$

and we shall assume that the coefficients A, B, . . . F have been derived by one of the usual methods.

Take rectangular axes SA, SB, and SC with reference to the plane of the true orbit, and let (ξ, η, σ) be the co-ordinates of the companion at any time with respect to these axes. Since the primary S is at a focus of the true orbit, the equation of the true ellipse is

$$\frac{(\xi + ae)^2}{a^2} + \frac{\eta^2}{b^2} = 1. \quad (2)$$

where

$$b^2 = a^2(1 - e^2).$$

Let (l_1, m_1, n_1) , (l_2, m_2, n_2) , and (l_3, m_3, n_3)

denote the direction-cosines of SA, SB, and SC respectively with reference to the axes SL, SM, and SK. Then drawing great circle arcs from each of A, B, and C in turn to L, M, and K, we can write

$$\begin{aligned} l_1 &= \cos AL, & m_1 &= \cos AM, & n_1 &= \cos AK, \\ l_2 &= \cos BL, & m_2 &= \cos BM, & n_2 &= \cos BK, \\ l_3 &= \cos CL, & m_3 &= \cos CM, & n_3 &= \cos CK, \end{aligned}$$

and from the appropriate spherical triangles we derive at once

$$\left. \begin{aligned} l_1 &= \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i \\ m_1 &= \sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i \\ n_1 &= \sin \omega \sin i \end{aligned} \right\} \quad (3)$$

$$\left. \begin{aligned} l_2 &= -\cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i \\ m_2 &= -\sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i \\ n_2 &= \cos \omega \sin i \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} l_3 &= \sin \Omega \sin i \\ m_3 &= -\cos \Omega \sin i \\ n_3 &= \cos i \end{aligned} \right\} \quad (5)$$

We require subsequently the following relations between the direction-cosines :—

$$l_1 m_2 - l_2 m_1 = n_3 \quad (6)$$

$$l_1^2 + l_2^2 + l_3^2 = 1 \quad (7)$$

$$m_1^2 + m_2^2 + m_3^2 = 1 \quad (8)$$

$$l_1 m_1 + l_2 m_2 + l_3 m_3 = 0 \quad (9)$$

4. Any point (ξ, η) on the true orbit projects into the point (x, y) on the apparent orbit. We have consequently

$$\begin{aligned}x &= l_1\xi + l_2\eta, \\y &= m_1\xi + m_2\eta,\end{aligned}$$

from which, using (6),

$$\begin{aligned}\xi &= \frac{m_2x - l_2y}{n_3}, \\ \eta &= -\frac{m_1x - l_1y}{n_3}.\end{aligned}$$

Substituting these values of ξ and η in (2), we obtain

$$\frac{(m_2x - l_2y + aen_3)^2}{a^2n_3^2} + \frac{(m_1x - l_1y)^2}{b^2n_3^2} = 1 \quad . \quad . \quad (10)$$

This last equation is the equation of the apparent orbit, and is therefore equivalent to (1). The coefficients of the same powers of the variables in (10) are therefore proportional to the corresponding coefficients in (1). Denoting the common ratio by f , we have, considering the coefficients of $x^2, y^2, \dots$ in turn,

$$\left. \begin{aligned}A &= \frac{f}{n_3^2} \left(\frac{m_2^2}{a^2} + \frac{m_1^2}{b^2} \right); & B &= \frac{f}{n_3^2} \left(\frac{l_2^2}{a^2} + \frac{l_1^2}{b^2} \right) \\ H &= -\frac{f}{n_3^2} \left(\frac{l_2m_2}{a^2} + \frac{l_1m_1}{b^2} \right) \\ G &= \frac{fel_2}{an_3}; & F &= -\frac{fel_2}{an_3}\end{aligned} \right\} \quad . \quad (11)$$

and from the absolute terms

$$1 = f(e^2 - 1)$$

or writing

$$p = a(1 - e^2) \equiv b^2/a \quad . \quad . \quad . \quad (12)$$

we have

$$f = -\frac{a}{p} \quad . \quad . \quad . \quad (13)$$

which can then be substituted in the expressions of (11).

5. We now derive the elements in terms of $A, B, \dots F$. The form of the expressions (11) for $A, B, \dots F$ immediately suggests the combination of F^2 and G^2 with A and B , and the combination of H with the product of F and G . We have, firstly, using (12) and (13),

$$\begin{aligned}F^2 - G^2 + A - B &= \frac{e^2}{p^2n_3^2}(l_2^2 - m_2^2) - \frac{a}{pn_3^2} \left\{ \frac{m_2^2 - l_2^2}{a^2} + \frac{m_1^2 - l_1^2}{b^2} \right\} \\ &= \frac{(l_2^2 - m_2^2)}{p^2n_3^2} \left(e^2 + \frac{p}{a} \right) - \frac{(m_1^2 - l_1^2)}{p^2n_3^2} \\ &= \frac{1}{p^2n_3^2} (l_1^2 + l_2^2 - m_1^2 - m_2^2) \\ &= \frac{1}{p^2n_3^2} (m_3^2 - l_3^2) \quad \text{by (7) and (8).}\end{aligned}$$

Hence using the values of l_3 , m_3 , and n_3 given by (5), we obtain

$$F^2 - G^2 + A - B = \frac{\tan^2 i}{p^2} \cos 2\Omega \quad . \quad . \quad (14)$$

Again,

$$\begin{aligned} FG - H &= -\frac{e^2 l_2 m_2}{p^2 n_3^2} - \frac{a}{p n_3^2} \left(\frac{l_2 m_2}{a^2} + \frac{l_1 m_1}{ap} \right) \\ &= -\frac{1}{p^2 n_3^2} (l_1 m_1 + l_2 m_2) \\ &= \frac{l_3 m_3}{p^2 n_3^2} \quad \text{by (9).} \end{aligned}$$

Hence, using (5),

$$FG - H = -\frac{1}{2} \frac{\tan^2 i}{p^2} \sin 2\Omega \quad . \quad . \quad (15)$$

From (14) and (15) we obtain

$$(F^2 - G^2 + A - B) \sin 2\Omega + 2(FG - H) \cos 2\Omega = 0 \quad . \quad (16)$$

This last equation determines Ω in terms of the known quantities A, B, . . . F.

The quantity $\frac{\tan^2 i}{p^2}$ can then be found from (14) or (15).

Again, it is easily seen that

$$\begin{aligned} F^2 + G^2 - A - B &= \frac{1}{p^2 n_3^2} (l_1^2 + l_2^2 + m_1^2 + m_2^2) \\ &= \frac{1}{p^2 n_3^2} (2 - l_3^2 - m_3^2) \\ &= \frac{1}{p^2 \cos^2 i} (2 - \sin^2 i) \\ \therefore F^2 + G^2 - A - B &= \frac{2}{p^2} + \frac{\tan^2 i}{p^2} \quad . \quad . \quad (17) \end{aligned}$$

Since $\tan^2 i/p^2$ has already been found, this last formula enables p to be calculated. The inclination i is then obtained.

Again, from (4) and (11),

$$-m_2 \equiv \sin \Omega \sin \omega - \cos \Omega \cos \omega \cos i = \frac{Gp \cos i}{e} \quad . \quad (18)$$

$$-l_2 \equiv \cos \Omega \sin \omega + \sin \Omega \cos \omega \cos i = -\frac{Fp}{e} \cos i \quad . \quad (19)$$

Multiply (18) and (19) by $\sin \Omega$ and $\cos \Omega$ and add. Then

$$e \sin \omega = p(G \sin \Omega - F \cos \Omega) \cos i \quad . \quad . \quad (20)$$

Multiply (18) and (19) by $\cos \Omega$ and $\sin \Omega$ and subtract. Then

$$e \cos \omega = -p(G \cos \Omega + F \sin \Omega) \quad . \quad . \quad (21)$$

Hence from (20) and (21)

$$(G \cos \Omega + F \sin \Omega) \tan \omega = - (G \sin \Omega - F \cos \Omega) \cos i \quad (22)$$

This equation, together with (20) and (21), determines ω .

When ω has been found, the eccentricity e is obtained from (20) or (21).

The last element with which we are concerned is the semi-major axis a ; it is found by means of

$$p = a(1 - e^2),$$

in which p and e have already been evaluated.

Observatory, Cambridge :

1930 February 24.

The Use of a Floating Mirror with the Transit Circle.

By H. C. Woods, Ph.D., B.Sc.

The object of this paper is to demonstrate the value of a mirror mounted so as to float upon mercury, as a means of observing the zenith and horizontal points directly with the transit circle, the nadir being already observable by reflection at a mercury surface.

Captain Kater in 1825 * made experiments with small collimating telescopes fixed on solid wooden and iron floats, 7 and 8 inches in length, floating in mercury. Observing the cross-wires of these telescopes with a fixed telescope he obtained results which showed that the float returned, after drastic disturbance, to the same inclination to the horizon with remarkable accuracy. Later † he invented a "Vertical floating collimator," which consisted of a small telescope with carefully made fiducial mark pointing downwards through the centre of an annular trough, rigidly mounted on a solid iron float, pivoted inside the trough by polished horizontal pins fitting in slots. He mounted the trough on rollers so that it could be accurately reversed, and obtained observations of the zenith with an altazimuth and a zenith telescope which he applied with success to determinations of latitude.

Since then the principle of flotation in mercury has been used by Chandler and others in the Almucantar, and by Bryan Cookson in the floating zenith telescope at present in use for observing variations of latitude at Greenwich.‡ It is estimated of the Durham Almucantar that after displacement it returns within $0''.05$ of its previous position. These instruments are of much larger size than that used in the present investigation.

In the recent instruments the method of eliminating the unknown inclination of the optical axis to the vertical has been to reverse the float within the trough. Captain Kater's vertical collimator was

* *Phil. Trans.*, 1825, p. 147.

† *Ibid.*, 1828, p. 257; see also *ibid.*, 1826, p. 307.

‡ *Monthly Notices R.A.S.*, **60**, 572, 1900; **61**, 315, 1901.

designed so that the trough and float were turned together, and in the present experiments the same method is adopted. The difficulty of designing suitable pivots for the float is greatly simplified when it is not necessary to make them capable of being thrown in and out of action, and no springs pressing on the float are required.

The vertical floating collimator in which a small telescope was used had the disadvantage of the great difficulty of obtaining optical conditions suitable for precise measurements. The use of a plane mirror of the full aperture of the object-glass of the observing telescope is, if the mirror is perfect, equivalent to the use of a floating collimator of the same size as the observing telescope, though the size of the whole instrument need not be excessive even for the largest transit instruments.

Description of Transit Circle.

The transit circle is the Draper's Company's Transit Circle at University College, London. The telescope is of $3\frac{1}{2}$ inches aperture and 44 inches focal length. The eye end is fitted with a right ascension micrometer which moves the whole set of cross-wires in the field. The mounting is on concrete piers resting on concrete foundations under the observatory. There is no apparatus for raising or reversing the instrument, and the collimation has to be obtained by reflection in mercury, the level being found with a striding-level, and applied to the reflection observations. Two lenses fixed in the north and south windows enable two meridian marks about 60 feet distant to be observed.

Accuracy in Time Determinations.

The time-signals sent out from different observatories throughout the world have shown that the determinations of time made by these observatories are subject to errors larger than were to be expected from errors in the timing of transits, and of a character which suggests that the cause might be inaccuracies of determination of instrumental constants. The two constants which can normally be determined without star observations are the level and collimation errors. It is usual to determine the collimation error with collimating telescopes, and the level error may either be determined independently by means of a striding-level, or by reflection in a bath of mercury at the nadir. The striding-level is subject to the limitations of sensitiveness and accuracy of the level bubble, and requires a correction for any possible inequality in the size of the pivots. There may also be an error of the nature of a level error, caused by flexure of the axis, of which the striding-level cannot take account.

Professor Sampson * has shown that daily observations at Edinburgh might be improved in many cases by rejecting the observations of level by reflection at the nadir and obtaining it instead from the transits themselves, the azimuth error being found from observations of the collimators, the fixity of which may be relied on from day to day.

* *Monthly Notices R.A.S.*, **82**, 193, 215, 1922 ; **85**, 560, 1925.

This method cannot completely eliminate the azimuth error, but is shown to smooth out irregularities in the determination of clock rate, and an extensive investigation by M. R. Madwar* leaves no doubt that erratic determinations of time are greatly reduced by rejection of the level error as ordinarily observed.

When the level error is determined by reflection at the nadir, a correction has to be applied for the collimation error. The error of such a determination of level includes the error of observation of the coincidence of wires and the error of determination of collimation. If it were possible to make an observation of the zenith with the same accuracy as the reflection observation at the nadir, then the uncertainty of a transit error at the zenith would amount only to the uncertainty of this observation. The two vertical reflection observations would determine the level and collimation together, the reading at the zenith being affected with the same collimation as at the nadir, while the displacement due to level would have altered sign. The level thus determined is the true level of the effective axis of the instrument, and the collimation is obtained in the vertical position of the telescope.

In order to compare the value of different methods of observation of level and collimation, suppose that N transits of stars have been observed at various declinations and the adopted level and collimation errors have been applied in each case. Then the clock error (Δt) and azimuth error (a) have to be found by means of the following equation, Δb and Δc being the errors of the adopted level and collimation respectively.

$$(1) \quad (a - T) = \Delta t + a(\sin \phi - \cos \phi \tan \delta) + \Delta b(\cos \phi + \sin \phi \tan \delta) + \Delta c \sec \delta.$$

The equations for a least-squares solution are :

$$(2) \quad \Sigma(a - T) = N(\Delta t + a \sin \phi) - a \cos \phi \cdot \Sigma \tan \delta + N\Delta b \cos \phi + \Delta b \sin \phi \Sigma \tan \delta + \Delta c \Sigma \sec \delta.$$

$$(3) \quad \Sigma(a - T) \tan \delta = (\Delta t + a \sin \phi) \Sigma \tan \delta - a \cos \phi \Sigma \tan^2 \delta + \Delta b \cos \phi \Sigma \tan \delta + \Delta b \sin \phi \Sigma \tan^2 \delta + \Delta c \Sigma \tan \delta \sec \delta.$$

Eliminating a from these equations the component ($\Delta t'$) of the clock error which depends on Δb and Δc only is given by

$$(4) \quad \Delta t' = -\Delta b \sec \phi + \Delta c \cdot P,$$

where

$$P = \frac{\tan \phi (\Sigma \tan \delta \Sigma \sec \delta - N \Sigma \tan \delta \sec \delta) - (\Sigma \tan^2 \delta \Sigma \sec \delta - \Sigma \tan \delta \Sigma \tan \delta \sec \delta)}{N \Sigma \tan^2 \delta - (\Sigma \tan \delta)^2}$$

If two equal groups of stars, one near the equator and the other near the pole, are used, P becomes $-(1 + \tan \phi)$, or -2.28 in latitude 52° . A somewhat lower value will be obtained in practice, however, since when a polar star is used the residual error of collimation ($\Delta c \sec \delta$) is wholly ascribed to azimuth error (by the term $-a \cos \phi \tan \delta$) and the

* *Monthly Notices R.A.S.*, **86**, 137, 1926.

constant term in the azimuth error is of opposite sign to the correction which should be applied for the residual collimation error. If five transits are observed in declinations -20° , 0° , 20° , 40° , 60° , the value of P is -1.06 , and therefore in latitude 52°

$$\Delta t' = -1.62\Delta b - 1.06\Delta c.$$

The following three methods may now be compared:—

I. Level and collimation errors determined by reflection at zenith and nadir. Let e_z , e_n be the errors of measurement of the zenith and nadir points respectively.

Then

$$\Delta b = \frac{1}{2}(e_z - e_n), \quad \Delta c = \frac{1}{2}(e_z + e_n),$$

i.e.

$$\Delta t' = -1.34e_z + 0.28e_n.$$

II. Collimation determined by means of horizontal collimators and the level by reflection at the nadir. Let e_c = the error of measurement of collimation.

Then

$$\Delta b = e_c - e_n, \quad \Delta c = e_c,$$

i.e.

$$\Delta t' = 1.62e_n - 2.68e_c.$$

III. Level obtained from the zenith and nadir observations and collimation from the horizontal collimators.

Then

$$\Delta b = \frac{1}{2}(e_z - e_n), \quad \Delta c = e_c,$$

i.e.

$$\Delta t' = -0.81e_z + 0.81e_n - 1.06e_c.$$

The above figures indicate the relative importance of each of the three determinations, namely, zenith and nadir points, and collimation. So far as the zenith and nadir observations are concerned, method III., which gives them equal importance, is the best.

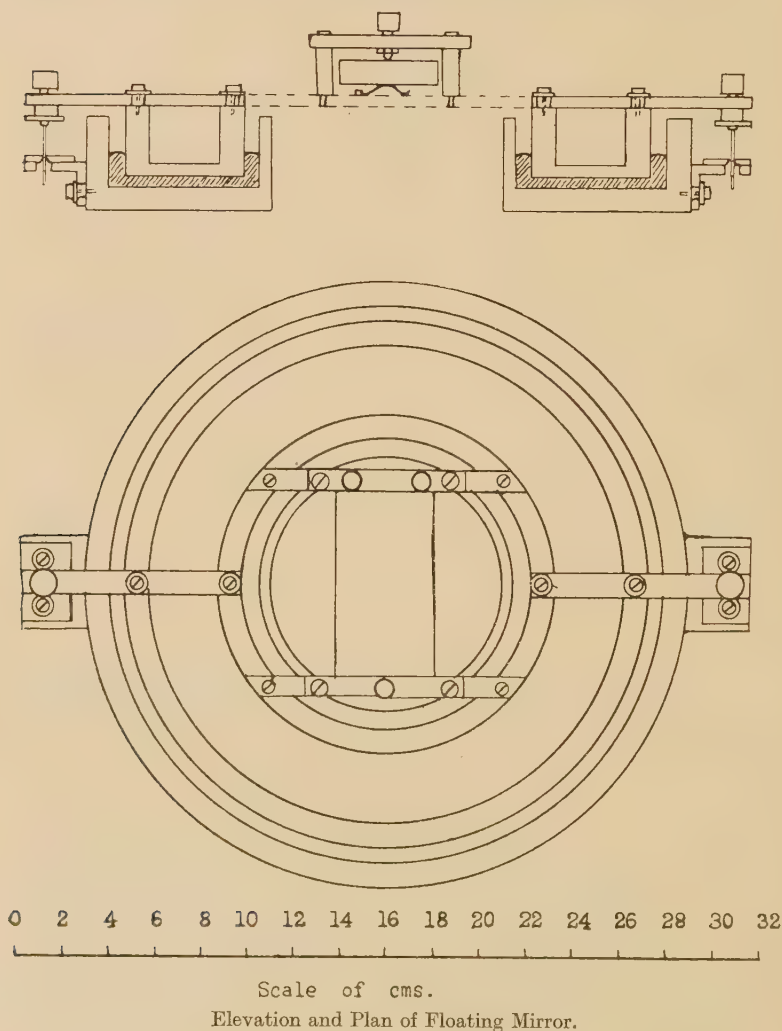
Description of Floating Mirror.

Preliminary experiments were made with a light wooden trough and ring, the arrangement being similar to that of the iron float and trough described below. This gave encouraging results and showed good accordance between mirror and mercury surface observations.

The mirror used is an optically plane parallel plate, made by Adam Hilger, Ltd., 1 cm. thick and having an available rectangular aperture $4\frac{1}{4}$ cms. by 8 cms. The wire images obtained with the mirror in collimation observations are good, having regard to the restriction of aperture, which affects the image seriously only for the wires at right angles to those being observed, the longer axis of the mirror being placed at right angles to the pivot axis of the float.

The wires are illuminated by means of a Bohnenberger eyepiece, and the setting is made with the R.A. micrometer screw, the value of one revolution of which is 4.477 secs. ($67''\cdot2$). The vertical wires

consist of six transit wires and a close pair about 20" apart in the centre. The settings in the observations referred to below were made so that the centre pair of wires were superimposed on the reflected pair.



The iron float and trough are made of cast iron turned to the dimensions shown in the diagram. The sides of the float and the outer side of the trough are 1 cm. thick, and the bottom of the trough the same. The bottom of the float and the inner side of the trough are $\frac{1}{2}$ cm. thick. The float is turned and polished on the sides and bottom. There is a free surface of mercury both inside and outside

the float of $\frac{3}{4}$ cm., and a clearance of $\frac{1}{2}$ cm. at the bottom. It required 10 lbs. of mercury to fill the trough to these dimensions.

On the outside of the trough at opposite ends of a diameter are two brackets of brass, each of which carries a brass plate drilled with a conical hole which lies over a larger hole in the bracket. These plates are held in position by bolts passing through holes in the plates drilled wide to allow of adjustment both for distance from the trough and laterally. The brackets themselves are also slotted to allow of vertical adjustment so that the narrow ends of the conical holes are in the level of the mercury surface.

Two brass arms are screwed to the top edges of the float to carry steel pins, 2 mm. thick, over the holes in the plates. These pins are held by passing through bolts screwed into the horizontal arms and secured with lock nuts. The conical holes are just large enough to allow the pins to shake slightly when in position. The arms carrying these pins are also slotted to adjust them so that the float is symmetrically placed in the trough.

Two pieces of flat brass rod are secured over the top of the float parallel to the axis of the pivots and with 8 cms. clearance between them. In the middle of each a similar short piece of brass rod is held on upright posts 2 cms. from the lower one. Through these pass the pointed screws against which the mirror is held by springs fixed to the lower rods. There is one screw on one of the frames and two on the other to afford an adjustable three-point suspension of the mirror. The adjusting screws are provided with lock nuts to ensure their complete rigidity. The pressure of the springs was made as light as possible in order to avoid distortion of the mirror, and the arrangement was found to give good definition of the reflected images with quite sufficient stability of adjustment.

Adjustments and Accuracy of Floating Mirror.

In estimating the accuracy of observations with the float, attention must be paid to the following points :—

(a) The absence of bubbles under the float or irregular surface tension of the mercury where it makes contact with the float, the complete freedom of the float to rotate about a horizontal axis, and the absence of all forces, apart from the effects of its own buoyancy and weight, tending to produce such a rotation.

(b) The rigidity of the mirror mounting.

(c) The steadiness of the float while observations are being made.

(d) The position of the axis of the float with respect to the plane in which the tilt of the mirror is to be measured, taking into account the tilt of the mirror in a vertical plane parallel to this axis.

(e) Optical conditions and accuracy of setting.

With regard to (a), the method of using the float does not require it to be removed from the mercury, and if the mercury is kept clean by wiping the surface from time to time, no difficulty should be experienced from particles of dirt on the sides of the float.

The resultant surface tension of the mercury as it affects the float has two components, one horizontal in the plane of the circle of contact, and the other vertical and displaced from the axis of the float to a distance depending on the irregularities of the tension at different points of the float. The existence of a horizontal component may be demonstrated by allowing the float to drift unconstrained in the trough, when it tends to be gradually drawn to the side. When the float is centred this component is very feeble and may be made harmless by placing the pivoting points as near as possible to the effective surface of the mercury. If the float beneath the surface makes clean contact with the mercury, the only disturbance of flotation is that due to the adhesion of the mercury round the float, causing a difference in the angle of contact from place to place. This adhesion will not be changed by small oscillations of the float, and in rotating the whole instrument the mercury flows under the surface film without causing any considerable change in the arrangement of the irregularities. It may be expected therefore that if the float after drastic disturbances of its position in the mercury returns with high accuracy to its original position, still greater precision should be attainable by careful handling in making the necessary reversals, preferably by placing the whole instrument on a ball-bearing.

The surface of the mirror being accurately plane and resting against three firmly fixed points, there should be no danger of movement altering the inclination to the horizontal. Temperature effects in the mounting may cause a slow movement. In this case the use of brass on an iron float would increase the effect, but it was found that holding the brass frame in the hand for a short time did not produce a noticeable displacement, so that in the interval between reversals no serious shift is likely, and a uniform rate of change produces no effect in a symmetrical set of observations.

The oscillations of the float are of two kinds, those due to the surging of the mercury round the trough, and superimposed on them the metacentric oscillations of the float itself. The iron float has a metacentric period of $\frac{1}{4}$ second approximately, and the longer oscillation is of 2-seconds period. After a disturbance of the float, if it is not subjected to further vibration, the metacentric oscillations die away in a few seconds, while the surging of the mercury may continue to produce evident displacements for some time. If the float has been only slightly disturbed, as when the trough has been rotated, the reflected images become sufficiently steady to make an observation in about half a minute. However, this observatory is situated within a few yards of a street (Gower Street) in which the traffic is constant and heavy, and it is very rarely that the oscillations are completely absent. The iron float does not vibrate with such liveliness as the wooden one, and it is not difficult to take advantage of quiet moments. It is, of course, absolutely essential to protect the instrument from draughts, as these might cause a steady tilt in the same direction and thus affect the determination of the zenith point.

If the accuracy of flotation were perfect, no matter how the axis

of the pivots might be oriented, the mean taken in R.A. or Declination would give the true vertical, provided the rotation of the trough between the two measures were exactly through 180° . But if the pivot axis, which for the first measurement was very nearly at right angles to the plane in which the tilt is measured, were on rotating inclined at a small angle α to its original direction, then any tilt which the mirror might have in the direction of the axis would have a component in the direction of measurement, while the tilt about the line of the pivots would still give approximately the same component. Hence an error $x \sin \alpha$ is introduced into the second measurement of tilt, where x is the tilt from the horizontal in the line of the pivots. If it is required to make observations with a maximum error of $0''.1$, the tilt in the line of the pivots must be less than $6''$ if the possible inaccuracy of rotation amounts to 1° . In practice this adjustment is easily made with a much smaller margin. The axis of the pivots in both positions should be as nearly as possible perpendicular to the direction of measurement, since though no error can be introduced if the flotation is perfect, the component of the tilt which is affected by the sliding of the pins is subject to error. This error was found to be very small with the iron float, and the adjustment therefore does not require much precision.

The shape and size of the mirror which has been used in this investigation introduces limitations to the optical perfection of the images obtainable by reflection. The longer axis of the mirror is always set so as to define best the wires being used, and since the clear aperture in this direction is 8 cms. (3.15 inches) no serious loss of definition results. The mirror must, however, be set centrally with regard to the telescope axis. The illumination of the wires is by means of a Bohnenberger eyepiece, a compound microscope of low power, before the object-lens of which is a diagonal plain glass reflector, reflecting part of the light of an electric lamp on to the cross-wires. This lamp must be placed so that the illumination completely fills the aperture in use; if this is not so, shifts of the wire images can be seen on moving it to and fro. With this eyepiece there are present in the field, both with the mirror and the mercury surface, a group of easily distinguishable interference fringes, six or seven in number, with a separation of about $20''$. They are in focus when the eyepiece is adjusted to show the cross-wires, and become indistinct on either side of this plane. The bands extend in a direction nearly perpendicular to that from which the illumination comes. Their centre is two or three bands away from that point in the field which is coincident with its reflection, and therefore in making a setting with the illumination from the side the central transit wire and its image are brought to coincide in the part of the field which is affected with the bands. A slight change in distinctness of the reflected images is observable as they pass over the bands, which must be considered a possible source of error. The location of the bands is, however, constant with regard to the wire which is being observed in coincidence with its own reflection. Interference fringes are theoretically present with a parallel plate reflector in the eyepiece (see *Journ. de Physique*, 1892, p. 414, M. A. Hurion, "Sur les franges visible dans un oculaire

nadiral"), but should vary in separation at different distances of the horizontal mirror from the object-glass of the telescope, the separation being infinite when the distance is equal to the focal length of the object-glass. The fringes observed with this instrument, however, do not fulfil this condition, but are nearly the same width in all positions of the mirror. In the case of a parallel glass reflector, each ray from the source of illumination is divided into two by the partial reflections from the two faces of the reflector. When this double ray is returned to the reflector each component is again divided, and of the four rays resulting two have described nearly the same length of path and are therefore in a condition to interfere. The fact that in general the angle of incidence on the reflector is not the same as the angle at which the rays left it causes the ray which undergoes double reflection the second time to interfere with that which was doubly reflected the first time, the path difference depending on the change of the angle of incidence. For the reflector used (2 mm. thick) these fringes should be too wide apart, even with the mirror in a favourable position, to be easily observable under this power, but a similar effect can be produced by an inclination to one another of the two faces of the reflector, so that the rays at the commencement of their path and at the end have different thicknesses of mirror to traverse. The linear displacement on the reflector of the points of departure and incidence is but little affected by the position of the mirror, and the cause of this optical defect in the present case must therefore be the prismatic form of the reflector, the direction of the fringes in the field being by chance nearly that which they would have in the other case.

With this eyepiece a small difference in focus is very appreciable, and the fact that the vertical and horizontal wires lie in slightly different planes makes it impossible to obtain sharp focus of the wires direct and reflected for both sets. The focus of the telescope is permanent, and a compromise must be made when making settings by reflection.

These optical defects of the instrument, besides producing apparent displacements of the wires, particularly the reflected images, may also cause a difference of aspect of the reflected and direct images. This difference may vary according to the mirror or mercury surface used as reflector, and it therefore requires to be shown that the floating mirror is capable of giving results identical with those which the mercury surface affords. Normally there is considerable vibration at this observatory, and this is especially difficult to remove when the mirror is mounted on the overhead beams to observe the zenith. It can be reduced to a fairly small amplitude, and settings are made so that the reflected wire is seen as much to one side as the other of the wire on which it is being set. This differs from the method in the case of the mercury surface, in which case the image is made faint by the vibrations but remains steady, the reflected wire at the moment of coincidence being just completely hidden.

The floating mirror with the iron float was compared at the nadir with the mercury surface. The trough was placed on a board under the telescope, and marks were made on the trough and on the board

by means of which the necessary reversals could be made with an error not exceeding 1° . Four observations were made in each position of the mirror, the float being depressed between each consecutive pair and set into oscillation. The measurements were made with the R.A. micrometer. Observations of the mercury surface consisted of at least five settings. The results were as follows :—

Mirror : 59 ^r .162	Mercury : before 59 ^r .167
	after 59 ^r .161
„ 59 ^r .182	„ after 59 ^r .182
„ 59 ^r .180	„ before 59 ^r .180
	after 59 ^r .179
„ 59 ^r .182	„ before 59 ^r .187
	after 59 ^r .181.

These observations are affected by changes taking place in the constants of the transit circle, and possibly by changes in the observer's personality in the setting, but there is no reason to think that any systematic difference exists between the two methods.

Method of Mounting Float at the Zenith.

Two oak planks, 1 inch thick and 11 inches wide, were mounted on their edges across the observatory on either side of the telescope. The ends rest on the walls of the observatory and the top edge is planed smooth. Across these beams is placed a wooden frame consisting of two boards fastened together just inside the beams by laths. The distance between the boards is sufficient to leave a clear aperture for the mirror. The trough is placed on a smaller smooth board, with a circular hole in it, mounted on levelling screws and placed across the opening in the wooden frame. The reversal of the trough was made by sliding it round on this board and setting marks on the board against marks on the trough. The distance between the beams is a little over 2 feet, almost exactly the same as the distance between the piers of the transit circle, thus leaving plenty of room for the observer to make the adjustments of the trough from a ladder. The frame is pushed to the end of the beams when not in use.

The vibrations to which the float is subjected in this position are severe. They are principally due to lateral impulses on account of the shape of the beams. If the beams are fixed by struts those impulses which cause slow and wide swinging of the float can be very largely removed, but the rapid oscillations which at times even blur the definition are increased. However, the amplitude of the vibrations was greatly reduced by placing on the beams under the four corners of the wooden frame small pads of soft rubber which easily yield to the lateral strains. When the impulses are momentarily absent the swing of the float takes place evenly from side to side and a setting can be made at the central point of the swing of the reflected wires.

The following observations were made with the floating mirror at the zenith. Seven observations were made consecutively in one position

(A) of the trough, each consisting of two micrometer settings, the float being disturbed after each observation, followed by a similar set in the other position (B), and repeated in the same order :

I.	A	58 ^r .683	.685	III.	A	58 ^r .667	.661
		.680	.689			.670	.670
		.688	.682			.671	.669
	Mean	.687	.680		Mean	.675	.672
	58 ^r .681	.676	.676		58 ^r .670	.671	.670
		.679	.679			.670	.670
		.680	.677			.669	.670
II.	B	59 ^r .147	.147	IV.	B	59 ^r .131	.140
		.149	.144			.130	.132
		.147	.144			.142	.137
	Mean	.148	.144		Mean	.137	.132
	59 ^r .145	.150	.150		59 ^r .135	.138	.138
		.140	.140			.134	.137
		.144	.138			.130	.132

The change in reading for both A and B positions is in the same direction and might therefore be the effect of a change in the instrumental constants such as affects also the mercury surface readings at the nadir, or it might be due to a slight steady draught in the observatory, though the roof was kept shut throughout. The differences in the two readings made between each pair of disturbances indicate how much of the variations should be ascribed to errors of setting, though there is distinct evidence of bias in the frequency with which the same reading is repeated. The average difference between these pairs of settings is $\pm 0^r.003$, and the average deviation of each observation from the mean of the group of seven is $\pm 0^r.0028$. This suggests that errors of flotation cannot be of very great importance in determining the zenith point, but it is clearly necessary to eliminate errors arising from vibrations if high accuracy is to be attained. The settings made at the zenith involve more risk of bias on the part of the observer on account of the uncertain nature of the observations, but where vibration is not present and a full aperture mirror is used, the results should be quite as reliable as those with the mercury surface, with which two sets of ten readings showed an average deviation from their means of $\pm 0^r.0018$.

Observations of Level, etc., with Floating Mirror.

The following observations were made with the iron float at the zenith. The level and collimation shown by the striding-level with the nadir observation and by the zenith and nadir method are compared, and the last column of the table shows the correction to the striding-level results on the supposition that the reflection observations of level are correct. Each observation of the zenith given below consisted of two settings of the float in position B interposed between two in position A. The float was disturbed between the B observations. In each of the four cases three readings of coincidence were made. The large value of the

level error on the first two days introduced an uncertainty of the scale value of the striding-level, this being nearly the extreme value observable. Between the two observations on June 3 the level of the transit circle was adjusted. Both methods indicated a change of collimation after this adjustment. Throughout this series of observations the tilt of the mirror to the horizontal in the line of the pivots was very small (not exceeding $2''$).

The collimation error seems to have altered between June 10 and 11, which is probably due to exposure of the telescope to strong sunlight before the observations on June 11. Apart from this the reflection observations give consistent results for the collimation, which is shown to have remained nearly constant from June 3 to 10 and 11 to 24. The striding-level shows much wider fluctuations. Since the level error has varied considerably over this period the changes of the nadir observation afford confirmation of the zenith results.

Date.	Zenith.	Nadir.	Collimation.		Level.		Correc- tion to S. Level.
			Ref.	S. Level.	Ref.	S. Level.	
1925.	r	r	r	r	s	s	s
May 29	58.819	59.188	59.003	59.028	-0.824	-0.714	-0.110
June 2	.789	.235	.008	.037	-1.010	-0.880	-0.130
	.778	.232					
" 3	.789	.241	.015	.060	-1.009	-0.810	-0.199
" 3	.999	.012	.006	.022	-0.031	+0.047	-0.078
" 4	59.019	58.985	.002	.013	+0.045	+0.127	-0.082
	.017						
" 5	58.972	59.026	.001	.023	-0.116	-0.013	-0.103
	.979						
" 8	.986	.022	.004	.019	-0.074	-0.013	-0.061
	.984						
" 9	.980	.039	.007	.017	-0.143	-0.100	-0.043
	.971						
" 10	.959	.057	.007	.015	-0.224	-0.187	-0.037
	.954						
" 11	.972	.071	.021	..	-0.223	..	..
" 12	.952	.089	.022	.039	-0.300	-0.226	-0.070
	.957						
" 22	.900	.140	.020	.039	-0.538	-0.453	-0.085
" 23	.917	.119	.017	.019	-0.457	-0.446	-0.011
	.913						
" 24	.930	.112	.021	.026	-0.398	-0.385	-0.013

Observations of a millimetre scale placed in the focus of the lens in the north window were made to test the micrometer, which had previously undergone repair for a large periodic error due to faulty bearings. It was found that a small error of the same kind was present,

but the maximum correction required was $\pm 0^{\circ}.005$. A correction has been applied for this error to the zenith observations and the accompanying nadir observations.

The striding-level is obviously not capable of much precision, since its readings vary greatly on the same day when the nadir remains constant. It was tested with a level tester, readings being made at intervals of from 5" to 2", as shown by the tester. The readings obtained as the tilt was varied in one direction differed systematically from those in which the tilt was altered in the opposite direction, the difference varying according to the interval between successive settings. Experiments were made placing the level on the transit circle in such a way that the bubble approached from one side or the other, as desired. It was found that in the ordinary use of the level it might be expected to approach from the East. Four sets of four readings each way were made with the following result :—

Bubble from E	.	Level Error =	$-0^{\circ}.434$
" " W	.	" "	$= -0^{\circ}.505$
" " E	.	" "	$= -0^{\circ}.393$
" " W	.	" "	$= -0^{\circ}.534$

The difference E-W is $+0^{\circ}.106$, and therefore the correction to the usual striding-level observations is $-0^{\circ}.053$, which is in quite good agreement with that indicated by the reflection observations.

The above reflection observations indicate a mean square error of a collimation determination by the zenith and nadir method of not more than $\pm 0^{\circ}.002$. Taking the same value for the mean square error of the nadir observation, then the mean square error of the zenith observation (σ_z) is given by

$$\frac{1}{4}(\sigma_z)^2 + \frac{1}{4}(0^{\circ}.002)^2 = (0^{\circ}.002)^2,$$

i.e.

$$\sigma_z = \pm 0^{\circ}.0035.$$

If the mean square errors of zenith and nadir observations are taken to be :

$$\sigma_z = \pm 0^{\circ}.0035,$$

$$\sigma_n = \pm 0^{\circ}.0020,$$

and the mean square error (σ_c) of collimation determinations by means of horizontal collimators be taken as $\pm 0^{\circ}.002$, then the effect on determinations of clock error of these errors of measurement in the three cases previously mentioned is as follows :—

I. $(\sigma_{\Delta t})^2 = (1.34\sigma_z)^2 + (0.28\sigma_n)^2,$

i.e.

$$\sigma_{\Delta t} = \pm 0^{\circ}.0047 \quad \text{or} \quad \pm 0^{\circ}.021.$$

II. $(\sigma_{\Delta t})^2 = (1.62\sigma_n)^2 + (2.68\sigma_c)^2,$

i.e.

$$\sigma_{\Delta t} = \pm 0^{\circ}.0062 \quad \text{or} \quad \pm 0^{\circ}.028.$$

$$\text{III. } (\sigma_{\Delta r})^2 = (0.81\sigma_z)^2 + (0.81\sigma_n)^2 + (1.06\sigma_c)^2,$$

i.e.

$$\sigma_{\Delta r} = \pm 0.0039 \quad \text{or} \quad \pm 0.017.$$

In practice, with better observing conditions, it may be expected that σ_z will be nearly equal to σ_n , which would give method I. an advantage over method II., but method III., in which the zenith and nadir observations are used together to determine the level only, seems the best all round where any uncertainty exists in the errors of the zenith observations, and provided σ_c is sufficiently small. Now the optical conditions in reflection observations have been shown to suffer from imperfections and therefore an error common to zenith and nadir observations is not improbable. In method III. no effect is produced by a systematic error in reflection observations, and errors in the collimation do not affect the level error as is the case when reflection is used only at the nadir. The choice between the methods I. and III. will depend upon the ratio of the errors of the two reflection observations, and on the relative magnitudes of the systematic errors to be expected in each of the two methods of determining collimation, as well as on the random errors of observations with the horizontal collimators. Method I. perhaps places too much reliance on the zenith reading, whereas method III. gives approximately equal importance to three independent observations, the errors common to zenith and nadir readings having no effect. It is probable that the above figure for the error of zenith observations should in fact be smaller, even for this observatory, and under better conditions all the errors σ_z , σ_n , σ_c might have a value of, say, ± 0.002 . In this case the effects on the clock error are :

$$\begin{array}{ll} \text{Method I. } \sigma_{\Delta r} = \pm 0.0027 = \pm 0.012, \\ \text{,, II. } \text{,,} = \pm 0.0062 = \pm 0.028, \\ \text{,, III. } \text{,,} = \pm 0.0031 = \pm 0.014. \end{array}$$

The mean square error due to errors of observation of instrumental constants is thus reduced by methods I. and III. to a value of the same order as that of the mean error in the timing of transits, and the only outstanding source of error is the systematic error unavoidably present in any determination of collimation, which is given much less prominence than in method II.

Other Uses of the Floating Mirror.

There are other uses to which the floating mirror may be put in connection with the transit circle. The horizontally placed mirror may also be used to obtain circle readings on the zenith, and if vertically mounted mirrors were placed north and south of the instrument, direct readings on the horizontal points might be made, giving in connection with zenith and nadir readings a means of determining possible differences of flexure in the north and south positions. If the mirror in this case were silvered on both sides the inclination of the two faces would have to be observed independently and would be a constant of the

instrument. The trough would be mounted so that it could be set accurately in azimuth and smoothly rotated.

If the mirror were inclined to the vertical the same method could be used to determine the horizontal point from observations equidistant above and below it, thus giving a value of the flexure at various zenith distances and affording a test of the law governing the R D discordance. The mounting would be more complex since the instrument would have to be raised and lowered as well as reversed, but it should be possible to effect this without serious disturbance, as in the case of the mercury bath used for reflection observations of stars. Unless its use is restricted to a small range the trough and float must be made so that the mirror could be seen on one side from below at all inclinations. Two rectangular troughs joined by a channel at one end should give the required accuracy. In these positions of the mirror the pointing of the float and mirror must be as exact as possible, but the shake of the pivots will impose a limit on this. However, the error in azimuth setting is indicated by the reflection of the vertical transit wires, and the small corrections which might be required can easily be applied.

Conclusion.

It is hoped that this paper has indicated a means of improving substantially the accuracy of time determinations with the transit circle by including a reflection observation of the zenith, thereby making the determination of level more symmetrical and independent of the collimation. The floating mirror used at University College has shown encouraging results as regards accuracy of flotation, and a permanent instrument adequately protected from vibration should give results equivalent to those of the mercury surface. It is shown that this method diminishes the importance of errors of coincidence settings in the final result for clock error, especially in the case of the collimation error. The author wishes to acknowledge his indebtedness to Professor Filon and Mr. Gregory for their advice and assistance throughout. Some criticisms by Dr. W. M. Smart have also been of great assistance.

Discussion of Observations of Three Long-period Variables, made by A. N. Brown, M.A. No. 2, V Cas., 1909-1929 January. By Mary A. Blagg.

(These observations, down to 1926 May, have been published in detail in *M.N.*, in four papers: 1910 June, 1916 May, 1921 May, and 1926 June.)

The discussion of these careful observations of V Cas., the second of the three stars undertaken, has again proved of great interest, an interest increased by its close resemblance in many respects to the first of the set, RT Cygni. As with the latter star, no actual maximum or minimum date can be fixed precisely by observation in any single period, for the

reasons given in the previous paper (*M.N.*, 1929 June). These stars appear to remain at a maximum or minimum magnitude, within the limits of accidental error, for about 10 or 12 days at least, and the British climate does not permit a sufficiently continuous and homogeneous set of observations from which to form a smooth and trustworthy curve in individual cycles. In this particular case, however, the fluctuations in the position of maximum and minimum are conspicuously apparent, and their investigation is thus more feasible than in the case of RT Cygni, where it proved better to study directly the relative variations of magnitude which accompany, cause, or result from, the fluctuations in the phases. Here the reverse is the case.

These fluctuations make the fixing of the star's mean period more difficult, and, unless the series available contains a full cycle of the fluctuations, it is evident that an *average* period will be pretty certain not to be the real "mean period" of the star.*

Inspection of these observations suggested a period of 228.6 days, and they were divided into cycles of that length, subdivided into 12 portions of 19.05 days each. This gave over 31 cycles in the twenty-one years. The mean magnitude was taken for each portion separately for (a) the first and second halves of the series, and also (b) for three divisions of 10 or 11 cycles each. By analysis, (a) gave a nominal correction of + 0.1 day to the assumed period, and (b) one of + 0.2 day. With an arrangement of cycles of 228.8 day a correction of nearly - 0.07 day was obtained, and from a trial of 228.72 days one of nearly + 0.07 day. (The fluctuations and the gaps prevent any very close agreement.) An assumed mean period of 228.76 days was then adopted for the working out of results. This gave no sensible correction. Table I. shows that it is not, in fact, the *average* period of the star's variation throughout the whole series. But, as is seen in Table II., the average length in different groups of cycles varies considerably in an obviously systematic manner. Whether 228.76 days is actually the star's true mean length of period is, of course, very doubtful, but there are certain indications that it approximates pretty closely to the most probable "mean period" so far as can be judged from these observations. No more than this is claimed for it, and it is used simply as a working basis in the investigation, *some* fixed mean period being a necessary starting-point.

The mean curve obtained from the whole of the observations worked out as follows (the starting-point of each first portion being J.D. 2418282 + 228.76):

Portions.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Mean } mag. }	7.98	8.50	9.48	10.60	11.56	12.16	12.38	11.98	10.88	9.73	8.78	8.10

* By an "average" period is here meant the simple average of a given number of consecutive cycles, which varies according to the particular series taken: by the "mean" period, that constant basic value which would form the average in a series containing an exact number of the periodic fluctuations. Thus, *e.g.*, the average daily duration of daylight for a few given months differs from that for other months, but the mean duration for any year or any complete number of years is constant.

Analysis of these figures gave the following formula :—

$$2.2 \cos (\theta - 169^{\circ}.7) + .04 \cos (2\theta - 76^{\circ}.5) + .06 \cos (3\theta - 233^{\circ}),$$

which, with the mean magnitude 10.18, forms the curve :

$$7.99, 8.49, 9.51, 10.61, 11.52, 12.17, 12.39, 11.95, 10.91, 9.73, 8.76, 8.13.$$

Not only is this curve in excellent agreement with the observed mean cycle, but it is remarkably like that found for RT Cygni (see *M.N.*, **89**, 688, 1929). The initial phase is different, depending as it does merely on the accidental starting-point, but when the first terms of all are brought into the same position the respective formulæ are :

$$\text{RT Cyg. : } 9.25 + 1.9 \cos (\theta - 180^{\circ}) + .05 \cos (2\theta - 98^{\circ}.8) + .05 \cos (3\theta - 24^{\circ}.2).$$

$$\text{V Cas. : } 10.18 + 2.2 \cos (\theta - 180^{\circ}) + .04 \cos (2\theta - 97^{\circ}.1) + .06 \cos (3\theta - 263^{\circ}.9).$$

The close resemblance in the first and second terms is striking and is borne out graphically by the circumstance that the two observed mean curves, when plotted, could well be mistaken for one another. In each case the rise is sharper than the fall, and in nearly the same proportion. RT Cygni has about 91 days rise to 100 days fall, and V Cas. nearly 108 days rise to nearly 121 days fall. The third term in each case is the most uncertain and may perhaps be variable. Its phase comes out differently in the two halves of the V Cas. series, but in both is in the same quadrant. The form of the mean curve is scarcely altered when a rather shorter or longer mean period is assumed.

As with RT Cygni, each observation was compared individually with the magnitude proper to its position in the mean curve, and the differences arranged in the same manner as before, using the signs “+” and “-” in their true arithmetical sense. There was only one striking instance of sudden great relative brightness, similar to those shown in Table I. for RT Cygni. It consisted of 5 consecutive residuals averaging - 0.9 m. in portions 11 and 12 of cycle 12, 1917 August and September. But there were four examples of great relative faintness : (1) In 1910 February, 4 consecutive residuals in portion 11 of cycle 1 averaged + 0.9. (2) In 1921 April and May, 7 consecutive residuals in portions 9 to 11 of cycle 19 gave an average of + 0.9 m. (This phenomenon began with an actual backward movement.) (3) In 1921-22 November to January, 11 consecutive residuals in portions 8 to 12 of cycle 20 gave an average of + 1.2 m. (4) In 1928 July, 4 consecutive residuals in portions 4 and 5 of cycle 31 gave an average of + 1.0 m.

But these phenomena occur less suddenly as well as less frequently than with RT Cygni, and, to a certain extent, they depend (in detail) upon the length of mean period assumed, especially those early or late in the series.

In order to look for possible “seasonal” variations, a table was drawn up of all the residuals, arranged according to months, in two divisions, 1909-18 and 1919-29 (January). Except for July, August, and November the means bore opposite signs in the two divisions. Those for July and August were negative and those for November

positive in both, as with RT Cygni, but very much smaller. In the totals, February to April and June to October were negative, and November to January and May were positive. But in all cases the figures were too small to be trustworthy, even had the two divisions shown less discordance. It seemed useless, therefore, to carry the investigation further, but one may conjecture that there is a slight tendency for this star also to appear relatively bright in the summer and relatively faint in the earlier winter months—probably from atmospheric and similar causes—while there seems no evidence of any systematic effect from configuration. Here also any error in the assumed length of period affects the results to a certain extent.

The next step was to investigate the fluctuations in the positions of maximum and minimum, which (as already stated) is here easier than to study the secondary variations in magnitude connected therewith. The dates of maximum and minimum suggested by the observer in the papers mentioned above are in most cases doubtful to about ± 4 days, and are sometimes still more uncertain, especially where gaps of many days occur near these points. Dates for maxima and minima have now been calculated by harmonic analysis of each individual cycle, this being done *separately* for maxima and minima. This process, in taking account of the whole cycle, minimises the effect of accidental errors (*e.g.* those due to atmospheric effects or the use of different comparison stars) in the particular observations round about maximum or minimum, but for the same reason it may swamp some real brief movement near those points. It may be regarded as giving the *mean* advance or retardation of the phases in each cycle, taking the normal positions of maximum and minimum alternately, as approximately the central points.

Table I. gives the results of these calculations. Column 1 gives the No. of the main period, adding "M" for maximum and "m" for minimum; column 2—"O"—gives the correction in days to the mean positions for these phases, obtained in the manner already explained, but here smoothed by taking the mean of each consecutive three corrections for maximum and minimum separately. This involves the rejection of the first and last of the corrections obtained. These figures are, of course, relative to the assumed mean length of period, but the dates and lengths of periods given in the next two columns are independent of this assumption. The mean position for maximum is taken as 2418285 + n 228.76, and that for minimum as 121 days later. The value of n at 1M is 2. (These mean positions, being read from the curve, are each subject to a possible *constant* error of anything up to about 1 day earlier or later.) If we take, instead of 228.76 days, a longer or shorter mean period (within reasonable limits) the corrections in the O column will automatically vary inversely with the altered mean position, and columns 3 and 4 will be unaltered. These dates differ, on the average, from those given in the observer's published papers by between 3 and 4 days, omitting cases in which a long gap near the special point makes his estimate particularly doubtful. In some cases it might be well to take the mean between the observer's estimates and these

TABLE I.

O Correction to Epoch. days	J. Date. 2410000+	Length of Period. 223d.+	Date from Calculated Fluctua- tions.	O Correction to Epoch. days	J. Date. 2420000+	Length of Period. 223d.+	Date from Calculated Fluctua- tions.
1M + 5.56	8748.08	..	8748.2	16M + 1.10	b2175.02	5.60	2174.2
m + 4.70	f8868.22	..	8869.0	m - 0.20	b(2294.72)	6.16	2295.5
2M + 3.93	b8975.21	4.14	8976.1	17M - 1.47	2401.21	3.19	2403.4
m + 2.66	9094.94	3.72	9096.0	m + 2.10	(2525.78)	8.06	2524.8
3M + 3.53	9203.57	5.36	9202.7	18M - 0.60	2630.84	6.63	2633.0
m + 1.70	(9322.74)	4.80	9322.3	m + 4.73	(2757.17)	8.39	2754.9
4M + 0.83	(9429.63)	3.06	9428.8	19M + 4.40	2864.60	10.76	2863.8
m - 0.60	(9549.20)*	3.46	9548.8	m + 5.90	vi2987.10	6.93	2986.1
5M - 1.94	9655.62	2.99	9655.8	20M + 8.76	3097.72	10.12	3095.2
m - 1.70	9776.86	4.66	9776.6	m + 7.26	vi3217.22	7.12	3217.2
6M - 3.24	(9883.08)	4.46	9884.4	21M + 10.36	3328.08	7.36	3325.8
m - 3.17	²⁴²⁺ (0004.15)	4.29	0005.6	m + 8.36	3447.08	6.86	3447.2
7M - 2.37	b(0112.51)	6.43	0113.7	22M + 9.63	3556.11	5.03	3554.5
m - 1.37	(0234.71)	7.56	0234.9	m + 8.13	3675.63	5.55	3674.7
8M - 1.14	0342.70	7.19	0342.7	23M + 7.00	3782.24	3.13	3781.1
m - 5.20	f(0461.64)	3.93	0463.3	m + 5.06	(3901.30)	2.67	3900.5
9M - 2.57	(0570.03)	4.33	0570.5	24M + 3.23	4007.23†	1.99	4006.6
m - 3.14	b(0690.46)	5.82	0690.6	m - 0.10	b4124.90	0.60	4126.0
10M - 4.50	0796.86	3.83	0797.2	25M - 2.40	b4230.36	0.13	4232.4
m - 4.60	(0917.76)	4.30	0917.4	m - 3.00	4350.76	2.86	4352.4
11M - 5.14	1024.98	5.12	1024.4	26M - 4.47	b4457.05	3.69	4459.3
m - 4.87	1146.25	5.49	1145.0	m - 3.10	f4579.42	5.66	4579.8
12M - 5.04	1253.84	5.86	1252.8	27M - 4.07	4686.21	6.16	4687.3
m - 3.87	1376.01	6.76	1374.4	m - 4.04	4807.24*	4.82	4807.9
13M - 4.07	vb1483.57	6.73	1483.1	28M - 4.74	b(4914.30)	5.09	4915.1
m - 2.00	f(1606.64)	7.63	1605.1	m - 4.60	5035.44	5.20	5035.3
14M - 0.60	1715.80	9.23	1714.1	29M - 6.50	5141.30	4.00	5142.3
m - 0.84	(1836.56)	6.92	1836.1	m - 7.77	5261.03	2.59	5262.4
15M + 1.26	f1946.42	7.62	1944.7	30M - 8.47	f5368.09*	3.79	5369.3
m - 0.60	(2065.56)	6.00	2066.3				

The average difference between columns 3 and 5 is a little less than ± 1 day.

* These three dates are considerably earlier than inspection of the observations alone would suggest. But when the cycles in question are plotted the discrepancy is seen to be due to an irregular depression or the reverse near the central point.

† This date would appear to be too late, but a similar remark applies to it.

calculated dates. Dates found for cycles much interrupted near the central points are enclosed in brackets in the table. The errors liable to be caused by gaps, etc. in the "corrections," probably sometimes amounting to at least ± 2 days, combined with the *general* approach to regularity in the figures, seemed to make the smoothing in column 2 legitimate and advisable. (The fluctuations were, however, first obtained from the unsmoothed figures.)

TABLE II.

Comparative Lengths of Periods : Group-averages.

Groups of 4.			Groups of 10.			Groups of 20.		
Cycles.	M.	m.	Cycles.	M.	m.	Cycles.	M.	m.
1 to 5	223+ 3·89	223+ 4·16	1 to 11	223+ 4·69	223+ 4·82	1 to 21	223+ 6·00	223+ 5·95
5 ,, 9	5·68	5·40	7 ,, 17	5·87	6·11	2 ,, 22	6·04	6·03
9 ,, 13	5·39	6·04	13 ,, 23	6·87	6·47	3 ,, 23	5·93	5·93
13 ,, 17	6·41	6·78	19 ,, 29	4·67	4·39	4 ,, 24	5·88	5·78
17 ,, 21	8·72	7·33	Average	5·52	5·45	5 ,, 25	5·74	5·70
21 ,, 25	2·57	2·92	Av. resid. $\pm 0\cdot84$	$\pm 0\cdot84$		6 ,, 26	5·70	5·76
25 ,, 29	4·74	4·57				7 ,, 27	5·68	5·63
Average	5·34	5·31	Groups of 14.			8 ,, 28	5·58	5·69
Av. resid. $\pm 1\cdot38$	$\pm 1\cdot23$		1 to 15	5·45	5·38	9 ,, 29	5·56	5·53
			8 ,, 22	6·53	6·57	10 ,, 30	5·56	
			15 ,, 29	5·20	5·25	Average	5·77	5·78
			Average	5·73	5·73	Av. resid. $\pm 0\cdot16$	$\pm 0\cdot13$	
			Av. resid. $\pm 0\cdot54$	$\pm 0\cdot56$				

An apparent fluctuation in about 20 periods was suggested both by the corrections to epoch and by the lengths of periods. The few examples given of variously grouped cycles in Table II. illustrate this, showing a clear fluctuation in each, until we come to those of 20 periods, when it is replaced by an almost steady small diminution, while these (20 periods) average group-lengths approximate closely to the adopted mean length of period. The gradual diminution shown (if not accidental) might be due to a secular shortening of period, or to the effects of additional longer or shorter fluctuations.* Against the first or second hypothesis is the fact that the longest periods occur late on, and only about 5 cycles before the shortest. The third hypothesis has been taken as the most probable solution, as there were other tokens of at least 2 shorter fluctuations, and the figures (both of the "corrections" and period-lengths) were analysed for various fluctuations round about 20, 12 or 13, and 6 or 7 periods. The three finally

* When longer groups than 20-period are taken, there is no such diminution shown. All that the said diminution actually proves is that cycles 2 to 9 are longer than cycles 22 to 29, individually (with one exception).

adopted were 20, $13\frac{1}{3}$, and $6\frac{2}{3}$ periods, in ratios 3, 2, and 1. These gave good agreement by analysis, and also, as may be seen in Tables I. and III., agreed in results with the figures obtained from the observations, within the limits of probable accidental errors. But it must be borne in mind that this series of observations is too short to allow of anything like real proof of (1) the true length of these fluctuations (especially the 20-period one), or (2) the precise mean length of period. In a sense, the adopted mean period and the length of the fluctuations are inter-dependent. If the mean period is really shorter, the fluctuations expressed in periods will be longer in proportion. Expressed in days they are (as adopted) 4575.2, 3050.13, and 1525.07. But, as already stated, these figures represent only a more or less plausible hypothesis, offered as a possible first step towards more definite conclusions which may be derived from further observations of this star in the future; and, at the best, the various figures suggested can be regarded only as approximations.

Formulae for these provisional corrections to epoch and fluctuations in period cannot conveniently be given apart from an assumed mean period. With a 228.76 mean period they are respectively

For maximum : $2418285 + 228.76 + 5.6 \cos(\phi - 24\frac{1}{2}^\circ)$
 $+ 1.67 \cos(3\phi - 142^\circ) + 1.67 \cos(1.5\phi - 276^\circ).$

For lengths of period : $228.76 \text{ days} + 1.75 \cos(\phi + 65\frac{1}{2}^\circ)$
 $+ 1.5 \cos(3\phi - 52^\circ) + 0.8 \cos(1.5\phi - 186^\circ),$

where ϕ advances 18° per 228.76 days, and equals 0° at 2418285 J.D.

TABLE III.

Period-lengths from Table I. : Means of M and m.

	O.	F ₁ .	O-F ₁ .	F ₂ .	F ₃ .	O-(F ₁₊₂₊₃).
	223+	223+				
1-2	3.93	5.02	-1.1	-0.2	-0.3	-0.6
2-3	5.08	4.57	+0.5	-1.3	+1	+1.7
3-4	3.26	4.23	-1.0	-1.5	+1.4	+0.1
4-5	3.83	4.04	-0.2	-0.3	+1.7	-0.6
5-6	4.38	4.03	+0.4	+1.0	+1.8	-1.4
6-7	7.00	4.18	+2.8	+1.5	+1.7	+0.6
7-8	5.56	4.48	+1.1	+0.7	+1.5	-0.1
8-9	5.08	4.91	+0.2	-0.6	+1.2	+0.6
9-10	4.06	5.45	-1.4	-1.5	-1.2	+0.3
10-11	5.31	5.97	-0.7	-1.1	-1.5	+0.9
11-12	6.31	6.50	-0.2	+0.2	-1.7	+0.3
12-13	7.18	6.95	+0.2	+1.3	-1.8	-0.3
13-14	8.08	7.29	+0.8	+1.4	-1.7	+0.1
14-15	6.81	7.47	-0.7	+0.3	-1.4	-0.6
15-16	5.88	7.50	-1.6	-1.0	-1.1	-0.5

TABLE III.—*continued.*

	O.	F ₁ .	O-F ₁ .	F ₂ .	F ₃ .	O-(F ₁ +2+3).
	223+	223+				
16-17	5.63	7.35	-1.7	-1.5	+3	-0.5
17-18	7.51	7.03	+0.5	-0.8	+6	+0.7
18-19	8.85	6.61	+2.2	+0.6	+8	+0.8
19-20	8.62	6.09	+2.5	+1.5	+8	+0.2
20-21	7.11	5.55	+1.6	+1.1	+6	-0.1
21-22	5.28	5.02	+0.3	-0.2	+3	+0.2
22-23	2.91	4.57	-1.7	-1.3	-1	-0.3
23-24	1.30	4.23	-2.9	-1.5	-4	-1.0
24-25	1.50	4.04	-2.5	-0.3	-7	-1.5
25-26	4.68	4.03	+0.7	+1.0	-8	+0.5
26-27	5.50	4.18	+1.3	+1.5	-7	+0.5
27-28	5.14	4.48	+0.7	+0.7	-5	+0.5
28-29	3.39	4.91	-1.6	-0.6	-2	-0.8
*(29-30)	(3.79)	(5.30)	(-1.5)	(-1.1)	(+0)	(-0.4)
Numerical total.			34.6			16.7
			* M.			

Owing to the doubtful character of the assumed mean period, it is not as yet feasible to deduce the magnitude waves, which necessarily accompany such fluctuations in phase and period, with any precision. The mean residuals, as read, in magnitude, when plotted for every sixth part of each mean period, show very plainly a wave of variation with an average length of just about 20/19ths of the assumed mean period corresponding to the 20-period fluctuation, but with variations in apparent amplitude and length due probably to other waves of variation. It does not disappear when corrections for a shorter mean period are roughly applied. The working out of its value and that of any other presumed waves cannot usefully be undertaken without a greater degree of certainty in regard to the mean length of period.

In Table III. the results of applying the above formula for variations in length of period are compared with the "O" figures. In this, as in the other tables, the fractional figures are not significant, but are retained for greater accuracy in taking averages and to avoid accumulated errors.

Summary.

1. A series of nearly 900 observations of V Cas., taken by A. N. Brown, M.A., extending over about twenty years, and comprising 31 periods (not including the first 5 scattered observations), have been tabulated and discussed.

2. A mean length of period of 228.76 days was found to suit these observations, and a smooth mean curve of variation was deduced, greatly resembling that of RT Cygni, and, like it, differing little from

a sine curve, but with the rise more rapid than the fall. Formulæ for the curve and for the mean dates of maximum and minimum are given.

3. The residual magnitudes, relatively to this curve, showed occasional great and sudden changes of magnitude at irregular intervals, but these were fewer and less marked than with RT Cygni. Little if any evidence of definite "seasonal" variations was found.

4. Corrections to phase (for maximum and minimum separately) were worked out in every cycle, and from these were deduced three waves of fluctuation in the phases and consequently in lengths of period. Formulæ for these are given, and their results are compared with the figures obtained from the observations. While the true length of the real mean period remains doubtful, these formulæ are given only as possible approximations to the truth, with a view to providing a starting-point for future investigations by which they may be partially confirmed, modified, or superseded.

5. The residual magnitudes, when plotted, showed a secondary wave of variation of about 20/19ths of the assumed mean period of main variation, which would correspond to the 20-period fluctuation found for the phases; but its true value cannot be profitably investigated without greater certainty as to the correctness of the assumed mean period.

*Preliminary Values of the Variation of Latitude at Greenwich
in 1928 and 1929.*

(Communicated by the Astronomer Royal.)

This paper contains the preliminary values of the latitude variation derived from observations made with the Cookson Floating Telescope.

The star places used were those derived from the final discussion of the observations for the period 1919-1926. As the mean zenith distance of some of the groups (each group containing 4 or 5 pairs) was becoming too large owing to precessional changes, 5 pairs were removed and 8 new pairs introduced.

The number of plates taken during 1929 was a record, but owing to cloudy weather there was only a comparatively small number of plates in April, October, and November.

The object-glass was removed and cleaned on October 31: after replacement the telescope was re-focussed and a new determination of scale made.

On measuring the scale plates taken in 1928 it was found that the scale had altered considerably in February of that year. We therefore include in this paper a revision of the latitude variation in 1928, using a revised scale value.

The variation, which has been very small for several years, increased considerably towards the end of 1929. The following table gives the

value of the latitude variation in the sense: mean latitude minus observed latitude :—

Date.	Variation.	Number of Pairs.
1928·171	+ ^u 066	109
·261	+030	71
·361	+0108	88
·437	-023	29
·505	-035	70
·574	-097	95
·668	-038	144
·776	-007	137
·900	+020	51
1929·060	-015	90
·167	+012	115
·263	-066	60
·366	+011	116
·450	-051	79
·492	-014	53
·554	-141	118
·659	-063	181
·764	+008	89
·926	+180	148
1930·047	+207	107

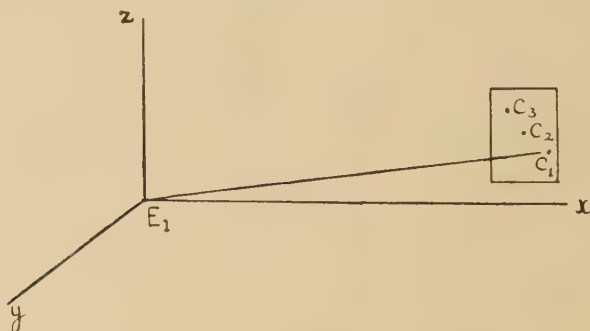
From a curve drawn to represent the observed results as closely as possible, the following corrections, applicable to observed north polar distances, were derived :—

1928·0	+ ^u 04	1929·0	+ ^u 02
28·1	+06	29·1	+03
28·2	+06	29·2	-02
28·3	+05	29·3	-03
28·4	+03	29·4	-03
28·5	-04	29·5	-09
28·6	-09	29·6	-10
28·7	-03	29·7	-04
28·8	00	29·8	+05
28·9	+03	29·9	+16
		1930·0	+20

A Mechanical Method for assisting in the Determination of Parabolic Orbits. By Rev. M. Davidson, B.A., M.Sc. (Plate 10.)

A mechanical model for the determination of the orbit of a comet can give, at the best, only a rough approximation. In the first place, it must not be on a large scale, as otherwise the position of the comet may be very high, especially if it has a large declination, and this renders the readings rather difficult—an experience I had with my first model. On the other hand, if the scale is too small, it may be impossible to be certain of accuracy beyond the first decimal place, and it is not worth the trouble to secure such a result.

After various trials I constructed a model with 1 metre as the astronomical unit and, on the whole, felt satisfied with the results. Before



describing it I shall deal briefly with the principles on which its working is based.

Let E_1C_1 be the line drawn from the centre of the earth to a comet, the R.A. and Decl. of which are α_1, δ_1 . If l_1, m_1, n_1 are the direction cosines of this line, then $\frac{x}{l_1} = \frac{y}{m_1} = \frac{z}{n_1}$. Let this line cut any plane, say $x = D$ or $y = D$ (a sketch of the relative positions of the sun, earth, and comet will show which is the more suitable); then if $x = D$,

$$y = D \frac{m_1}{l_1}, \quad z = D \frac{n_1}{l_1}.$$

Since

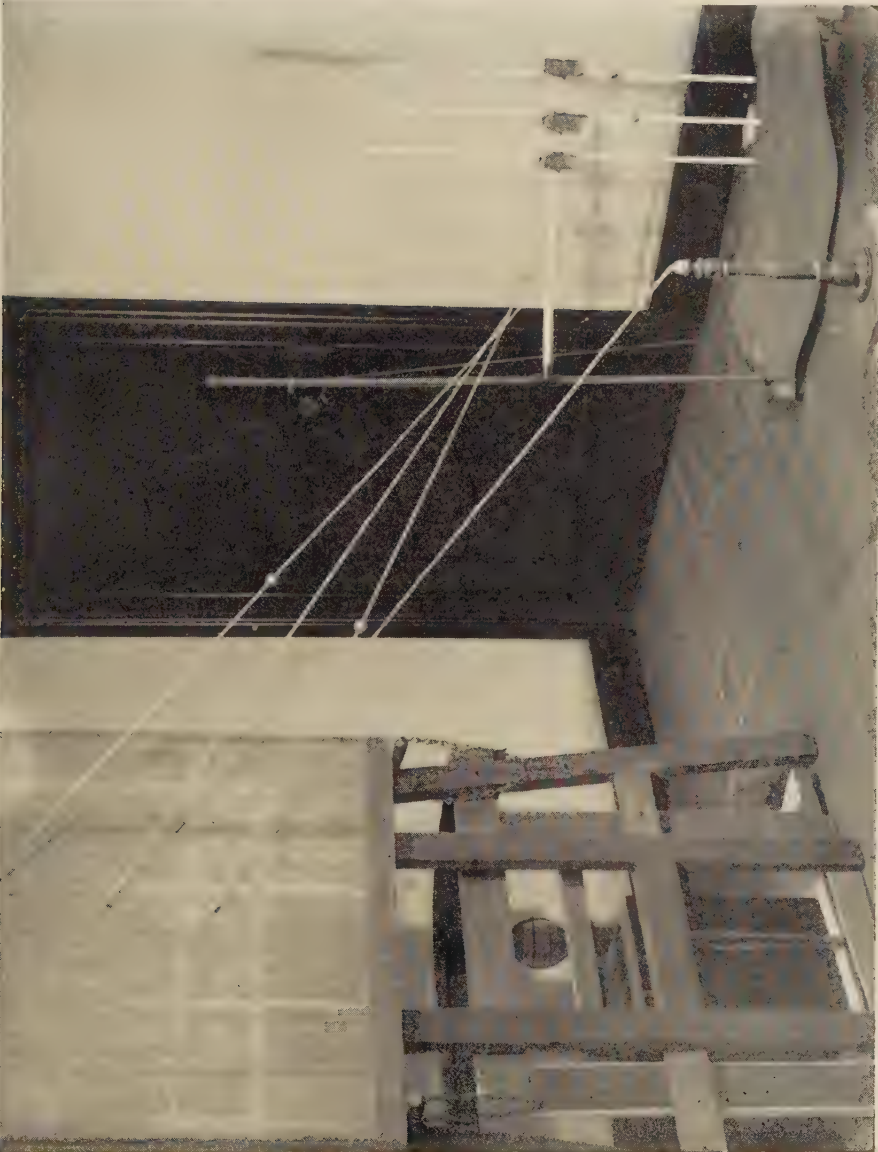
$$l_1 = \cos \delta_1 \cos \alpha_1, \quad m_1 = \cos \delta_1 \sin \alpha_1, \quad n_1 = \sin \delta_1,$$

the point on this plane will be determined by

$$x = D, \quad y = D \tan \alpha_1, \quad z = D \tan \delta_1 \sec \alpha_1.$$

If the solar equatorial co-ordinates at the time are X_1, Y_1, Z_1 , the heliocentric co-ordinates of the point will be

$$D - X_1, \quad D \tan \alpha_1 - Y_1, \quad D \tan \delta_1 \sec \alpha_1 - Z_1.$$



We can deal with the two other positions of the earth and comet in the same way, but it will be convenient to retain the plane $D - X_1$ expressed in heliocentric co-ordinates. Hence if the solar equatorial co-ordinates at the other times are X_2, Y_2, Z_2 , and X_3, Y_3, Z_3 , our second plane in geocentric co-ordinates will be $D - X_1 + X_2$, and our third one $D - X_1 + X_3$. The points where E_1C_1, E_2C_2, E_3C_3 cut the plane $D - X_1$ are marked on a large drawing-board bolted to a movable stand which can be placed in any desired position. A brass rod with a fine apex represents the sun, and to this there is attached a graduated wire, taking 1 metre as the astronomical unit. Supported by two uprights is a segment of a circle of radius 1 metre along which three pieces of steel tube can move and on which they can be secured by set-screws. From each of these steel tubes a brass rod projects vertically, and on each rod is a slide which can move with gentle friction. A thread is attached to a mark on each slide, and this mark is set for the value of Z with sign reversed, the top of the segment of the circle representing the equator. The positions of X and Y are found by measuring on the floor, two lines intersecting at right angles vertically under the sun being selected for axes. On the threads connecting the positions of earth and comet are very minute beads* which can slide about and which represent the positions of the comet. Of course the X and Y measurements are made, like the Z measurement, after reversing the sign of the solar equatorial co-ordinates.

If r_1, r_3 are two radii vectores from the sun to a comet in two positions differing by an interval of $t_3 - t_1$ days, Lambert's equation gives

$$(r_1 + r_3 + x)^{\frac{3}{2}} - (r_1 + r_3 - x)^{\frac{3}{2}} = 6k(t_3 - t_1),$$

assuming parabolic motion, where x is the chord joining the two positions and k is the Gaussian gravitation constant. For the purpose of the model this can be written in the form

$$(2r_2 + x)^{\frac{3}{2}} - (2r_2 - x)^{\frac{3}{2}} = 6k(t_3 - t_1),$$

where $2r_2 = r_1 + r_3$, or

$$(2r_2)^{\frac{3}{2}} \left[\left(1 + \frac{x}{2r_2} \right)^{\frac{3}{2}} - \left(1 - \frac{x}{2r_2} \right)^{\frac{3}{2}} \right] = (2r_2)^{\frac{3}{2}} \cdot \frac{3x}{2r_2} = 6k(t_3 - t_1).$$

This becomes $xr_2^{\frac{1}{2}} = k\sqrt{2}(t_3 - t_1)$

or, for the purpose of our calculations,

$$x = \frac{.02433}{r_2^{\frac{1}{2}}}(t_3 - t_1), \quad \text{since } k = .0172021. \quad (1)$$

It seems from this a simple matter to move the beads along two of the threads, say the first and third, and, by measurement, satisfy the above condition. As, however, x is not necessarily the shortest distance between the threads, there can be an infinite number of solutions and another condition is required.

* These are very much enlarged in the photograph, otherwise they would be invisible.

The next step is to discover a relation between the distances of the comet from the earth at the two periods. We do not propose utilising the formulæ* given in works dealing with the orbits of comets but shall make the model serve. Select any point on the middle thread as far as possible from E_2 and place a scale across this point, which we shall denote by C_2 , so that it runs across the other threads. If we imagine for the moment that $t_2 - t_1 = t_3 - t_2$, then the scale should be turned around C_2 so that, if it touches the other threads at C_1 and C_3 , $C_2C_1 = C_2C_3$ nearly. When this is done it will be found that on measuring C_3E_3 , C_1E_1 , the ratio is nearly the same as ρ_3 to ρ_1 , where ρ_3 , ρ_1 are the distances of the comet from the earth at the first and third observations. (It is important to notice that C_1 , C_2 , C_3 lie on a plane passing through S .)

This is not, of course, sufficiently accurate, and the following is the next best step for dealing with this point. Let $\frac{t_2 - t_1}{t_3 - t_2} = a$, which will generally differ very little from unity. Taking the point C_2 as before on any convenient part of the middle line, run the scale across so that $\frac{C_2C_1}{C_2C_3} = a$, and mark C_1 , C_2 , C_3 with beads. From the sun stretch the graduated wire and measure SC_1 , SC_2 , SC_3 . Suppose these are r_1 , r_2 , r_3 , then from (1) we see that C_1C_2 is proportional to $(t_2 - t_1)/(r_1 + r_2)^{\frac{1}{2}}$, and also that C_2C_3 is proportional to $(t_3 - t_2)/(r_2 + r_3)^{\frac{1}{2}}$, so that

$$\frac{C_2C_1}{C_2C_3} = \frac{t_2 - t_1}{t_3 - t_2} \cdot \left(\frac{r_2 + r_3}{r_1 + r_2} \right)^{\frac{1}{2}} \quad (2)$$

This ratio can be determined after measuring r_1 , r_2 , r_3 carefully, and then, by setting the scale across the threads and finding where C_1 and C_3 lie, the ratio C_3E_3 to C_1E_1 can easily be obtained.

We shall illustrate the method by an example from Dr. Merton's paper, *M.N.*, 85, p. 713, 1925, selecting Comet 1925 c (Orkisz).

A plane $x = 2$ was taken, and from $y = D \tan \alpha$, $z = D \tan \delta \sec \alpha$, we have $y = -.862$, $z = .645$. As the solar equatorial co-ordinates were then $X = -.968$, $Y = .233$, $Z = .101$, the heliocentric co-ordinates of the point where E_1C_1 intersects the plane are

$$x = 1.032, \quad y = -1.095, \quad z = .544.$$

The other points were found in the same way, the plane $x = 1.032$ referred to the sun, being retained, and then, the positions of the earth having been found on the slides, as previously described, these were connected by threads to the three points on the drawing-board.

The following are the actual figures obtained: we have $t_3 - t_1 = 9$, $t_2 - t_1 = t_3 - t_2 = 4.5$, then

$$x = \frac{.02433}{r_2^{\frac{1}{2}}} (t_3 - t_1) = \frac{.2189}{r_2^{\frac{1}{2}}}.$$

* For instance: J. Bauschinger, *Die Bahnbestimmung der Himmelskörper*, chap. xxii., p. 367; Bengt Strömgren, *Formeln und Tafeln zur Bestimmung parabolischer Bahnen*, p. 42; J. C. Watson, *Theoretical Astronomy*, pp. 171-3, 357.

Also, as $\frac{t_2 - t_1}{t_3 - t_2} = 1$, in the first determination of C_1 and C_3 to find the ratio ρ_3 to ρ_1 , $C_2C_1 = C_2C_3$. Marking C_1 , C_2 , C_3 on the threads, measurements gave as follows:

$$r_1 = 1.548, \quad r_2 = 1.514, \quad r_3 = 1.485.$$

Hence

$$r_1 + r_2 = 3.062, \quad r_2 + r_3 = 3.000,$$

so that

$$C_1C_2/C_2C_3 = \left(\frac{3}{3.062}\right)^{\frac{1}{2}} = .99.$$

A few trials gave the positions of C_1 and C_3 satisfying this condition, and on measuring to the earth it was found that

$$C_1E_1 = 2.241, \quad C_3E_3 = 2.078, \quad \text{or} \quad \rho_3/\rho_1 = .93 \text{ nearly.}$$

The next step was to take any point on E_1C_1 and by measurement find an approximation to the equation $x = \frac{.2189}{r_2^{\frac{1}{2}}}$. It is quite easy to find the limits for ρ_1 almost immediately by stretching the scale from the sun across the threads. It was obvious that $\rho_1 = 1.8$ was too great and $\rho_1 = 1.5$ too small, so $\rho_1 = 1.6$ was taken, ρ_3 found from $\rho_3/\rho_1 = .93$, r_1 and r_3 measured and then (1) applied. Afterwards $\rho_1 = 1.7$ was taken. The results are given below, where x_d means the chord C_1C_3 determined from $x = \frac{.2189}{r_2^{\frac{1}{2}}}$, that is, dynamically, and x_g denotes C_1C_3 measured with the scale—that is, found geometrically.

ρ_1	1.600	1.700
ρ_3	1.488	1.581
r_1	1.005	1.085
r_3	1.030	1.097
$r_2 = \frac{1}{2}(r_1 + r_3)$	1.017	1.091
x_d	0.217	0.209
x_g	0.193	0.205
$x_d - x_g$	0.024	0.004

It appeared that $\rho_1 = 1.7$ nearly satisfied the conditions but was too small, and also that a change of 0.1 in ρ_1 meant a change of .020 in $x_d - x_g$. Since $x_d - x_g$ should vanish, we can infer, as an approximation, that ρ_1 should be increased by $\frac{.004}{.020} \times .1 = .02$.

Hence $\rho_1 = 1.72$ is a nearer value for ρ_1 .

Using $\rho_1 = 1.72$ we found as follows:

$$\begin{array}{lll} \rho_1 = 1.72, & \rho_3 = 1.600, & r_1 = 1.100, \quad r_3 = 1.110, \\ x_d = .208, & x_g = .209, & x_d - x_g = -.001. \end{array}$$

It was impossible to obtain closer readings than these, and the measure-

ments were then taken for the middle bead, by placing it in a line with the others, giving $r_2 = 1.105$, $\rho_2 = 1.655$.

Before commencing the work to secure accurate results by Dr. Merton's method, these different values of ρ could be used to correct for light time. This, however, is scarcely necessary (though it can be done if desired), as Dr. Merton's method provides a simple means for making this correction at a later stage. The chief advantage of this approximate solution is to facilitate computations for a parabolic orbit which we shall now illustrate from the same example, referring to pp. 722-3 in Dr. Merton's paper.

The following formulæ are arrived at in the ordinary way in the paper (see p. 714):—

$$r_1^2 = \rho_1^2 - 1.58452\rho_1 + 1.00123. \quad (3)$$

$$\rho_3 = .94035\rho_1 + \Delta\rho. \quad (4)$$

$$r_3^2 = \rho_3^2 - 1.44513\rho_3 + 1.00639. \quad (5)$$

$$\Delta\rho = -.022802 + .023083\frac{1}{r_2^3}. \quad (6)$$

$$x_3 - x_1 = -.093788\rho_1 + .8365\Delta\rho + .048108,$$

$$y_3 - y_1 = +.078759\rho_1 - .3202\Delta\rho - .134514,$$

$$z_3 - z_1 = +.134172\rho_1 + .4448\Delta\rho - .058347.$$

At each step the approximation for r_2 is improved by

$$r_2 = c_1^0 r_1 + c_3^0 r_3 - c_1^0 c_3^0 \frac{r_2^2 - r_2(r_3 - r_1)^2}{2r_2^2}. \quad (7)$$

First, we shall start with $r_2 = 1.105$, and $\rho_1 = 1.720$ as found by the model. Then

$$r_1 = 1.11098, \quad \text{also } \Delta\rho = -.022802 + \frac{.023083}{1.34923} \text{ from (6),}$$

or

$$\Delta\rho = -.022802 + .017108 = -.005694.$$

$\therefore \rho_3 = .94035\rho_1 + \Delta\rho = 1.617367 - .005694 = 1.611673$ from (4), and $r_3 = 1.12907$ from (5). At this stage improve r_2 , using these values of r_1 and r_3 , and we find $r_2 = 1.11760$ from (7).

We can write the above expressions for $x_3 - x_1$, etc., as follows :

$$x_3 - x_1 = -.093788\rho_1 - .019074 + .019309\frac{1}{r_2^3} + .048108,$$

$$y_3 - y_1 = +.078759\rho_1 + .007301 - .007391\frac{1}{r_2^3} - .134514,$$

$$z_3 - z_1 = -.134172\rho_1 - .010142 + .010267\frac{1}{r_2^3} - .058347,$$

or

$$x_3 - x_1 = -\cdot 093788\rho_1 + \cdot 029034 + \cdot 019309\frac{1}{r_2^3},$$

$$y_3 - y_1 = +\cdot 078759\rho_1 - \cdot 127213 - \cdot 007391\frac{1}{r_2^3},$$

$$z_3 - z_1 = +\cdot 134172\rho_1 - \cdot 068489 + \cdot 010267\frac{1}{r_2^3}.$$

Use the value of r_2 just found, $r_2 = 1\cdot 11760$, or $r_2^3 = 1\cdot 3959$, and also $\rho_1 = 1\cdot 720$, in the above, and we have as follows:

$$x_3 - x_1 = -\cdot 161315 + \cdot 029034 + \cdot 013833 = -\cdot 118448,$$

$$y_3 - y_1 = +\cdot 135465 - \cdot 127213 - \cdot 005294 = +\cdot 002958,$$

$$z_3 - z_1 = +\cdot 230776 - \cdot 068489 + \cdot 007355 = +\cdot 169642.$$

$$s^2 = \Sigma(x_3 - x_1)^2 = \cdot 014030 + \cdot 000009 + \cdot 028778 = \cdot 042817,$$

or

$$s = \cdot 206923.$$

This s corresponds to our x_g , therefore $x_g = \cdot 206923$.

Since

$$r_1 + r_3 = 2\cdot 24005 \quad \text{and} \quad \eta = \frac{2\tau_2}{(r_1 + r_3)^{\frac{3}{2}}} = \frac{\cdot 30966}{(2\cdot 24005)^{\frac{3}{2}}} = \cdot 092.$$

$$\therefore \log \mu = 10\cdot 0001536$$

and

$$s = \mu \frac{2\tau_2}{(r_1 + r_3)^{\frac{1}{2}}} = \cdot 206965 \quad \text{or} \quad x_d = \cdot 206965.$$

Hence $x_d - x_g = \cdot 000042$.

From our model we found that if $x_d - x_g$ is positive we must increase ρ_1 , and that a change of $0\cdot 1$ in ρ_1 altered $x_d - x_g$ by $0\cdot 20$. Hence we can assume that ρ_1 should be increased by $\frac{\cdot 000042}{\cdot 020} \times 1$, that is, by $0\cdot 00021$.

A very close value of ρ_1 is therefore $1\cdot 72 + 0\cdot 00021 = 1\cdot 72021$. This agrees very well with Dr. Merton's value, $1\cdot 72027$.

It is interesting to notice how closely these values for x_g and x_d correspond to those obtained by measurement. It seems that the solution for a parabolic orbit can be considerably simplified by finding approximate values of ρ_1 and r_2 from the model, and using these in the usual equations to derive the accurate values. One substitution may suffice, and certainly not more than two should be necessary if the measurements are made accurately on the model.

Irrespective of the utility in finding a rough solution, the model is interesting in showing the reasons for the various stages in the computation of an orbit and also the difficulties in certain circumstances in determining the elements with great accuracy.

*On the Relation between the Geocentric and Heliocentric Velocities
and the Heights of Fireballs.* By V. A. Maltzev.

(Communicated by the Secretaries.)

1. The purpose of the present investigation, based on materials obtained from the *Katalog der Bestimmungsgrossen für 611 Bahnen grosser Meteore*, by G. v. Niessl and C. Hoffmeister,* is to establish a relation existing between the heights at which fireballs appear and disappear and their velocities. It should be noted that essentially the work was completed in 1926, and that at the time the author had no knowledge of Niessl's work connected with the subject.† As the material which we used was considerably vaster than that on which Niessl's work was grounded, and as we have succeeded in drawing certain new conclusions, we believe it to be of interest to publish the results obtained by elaborating Niessl-Hoffmeister's catalogue.

2. Recently the data presented in the catalogue were subjected to criticism in W. J. Fisher's paper;‡ the latter formed the conclusion that the majority of geocentric velocities, which had led to hyperbolic heliocentric velocities, were erroneous. Without entering into a detailed consideration of Fisher's paper, we shall point out a few insufficiently established statements which served as basis to his conclusions.

(a) Fisher states, that according to five different fireball catalogues there is one and the same periodicity in the number of observed fireballs both parabolic and hyperbolic. He says: "No one can doubt that the correspondence in figures 1 to 4, based on observations made, not only independently, but also by different methods, and during periods widely separated in time, are due to actual periodicities in the arrival of fireballs." Fisher particularly emphasises two maxima occurring on the 221-225th days of the year (very high maximum), and on the 316-320th days (high maximum). If Fisher had consulted the usual calendar dates he would have instantly noticed that these maxima, occurring on August 9-13 and on November 12-16, correspond to the periods when the observations on the Perseid and the Leonid showers are being carried out. Just on these days observers watch the sky more closely than at any other time, and therefore it is quite evident that the largest numbers of registered fireballs correspond to these days. Moreover, Fisher himself remarks that the fireball radiants are widely scattered during the Perseid epoch and show little sign of clustering, which only confirms the fact that the maxima occurring during these days should be attributed to the increased number of observers, but not to the earth's meeting with some fireball cluster.

(b) After considering the distribution of fireball radiants over the

* *Denkschriften d. Akademie d. Wissenschaften in Wien, mathemat.-naturwissensch. Klasse*, Bd. 100, Wien, 1925.

† *Encyklop. d. math. Wissensch.*, vi (2), 10, pp. 454-459, 1910.

‡ W. J. Fisher, "Remarks on the Fireball Catalogue of von Niessl and Hoffmeister," *Harv. Circ.*, 331, 1928.

celestial sphere, Fisher forms the following conclusion: "Certain fireball radiants cluster both in celestial position and in time about well-known shower radiants; so that the hyperbolic velocities of these clusters must be mostly erroneous" (my italics). A number of astronomers had already given their attention to the coincidence existing between fireball radiants and meteor radiants, and this led them to believe that a considerable number of meteor streams are also possessed of hyperbolic velocities (Kleiber,* v. Niessl,† Olivier,‡ and others). Niessl even investigated in detail several hyperbolic clusters of fireballs. Recent investigations carried out by C. Hoffmeister, N. Staude, V. Fessenkoff, and others also speak in favour of the meteors being partly possessed of hyperbolic velocities. Thus the above-presented conclusion drawn by Fisher, for which his article holds no proofs and which contradicts the generally accepted view, can by no means

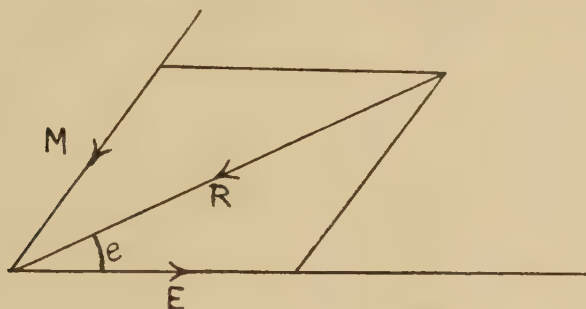


FIG. 1.

be considered as well grounded. Fisher ought to have proved that annually-recurring hyperbolic streams cannot exist.

(c) In his paper Fisher advances certain conditions under which an error in the geocentric velocity will produce an error of opposite sign in the heliocentric velocity. However, Fisher's essential equation,

$$M^2 = E^2 + R^2 + 2RE \cos e \quad (1)$$

(where M is the heliocentric velocity, R the observed velocity of the meteor, E the earth's orbital velocity, e the apparent elongation), contains an error in the last sign. According to the well-known trigonometrical formula it should be:

$$M^2 = E^2 + R^2 - 2RE \cos e \quad (1')$$

Therefore Fisher's further conclusions are erroneous. On accepting the correction of formula (1') we have:

$$\frac{\partial M}{\partial R} < 0 \quad \text{provided that} \quad \cos e > \frac{R}{E}.$$

* J. A. Kleiber, *The Astronomical Theory of Shooting Stars*, St. Petersburg, 1884, pp. 74-75 (in Russian).

† G. v. Niessl, *op. cit.*, p. 461.

‡ C. P. Olivier, *Meteors*, pp. 264-272, Baltimore, 1925.

Fisher found that for the 25 fireballs which fulfilled the conditions (a) $90^\circ < e < 180^\circ$, and (b) $R < E/\cos e$, their mean heliocentric velocity ($M_0 = 47.1$ km./sec.) was greater than the earth's orbital velocity ($E = 29.5$ km./sec.): this contradicted the inequalities he had deduced and according to his belief pointed towards the erroneousess of the heliocentric velocities. But the inequality $M_0 > E$ may easily be obtained from formula (1') if e fulfils the condition (a). (It should be remembered that M , R , E are essentially positive.) The fact that Fisher obtained results which contradicted theory when he compared them with observations only shows the erroneousess of his conclusions, and by no means that of the material he used.

(d) Fisher refers to the fact that the only reliable meteor velocities which have been photographically determined by Elkin show but very small heliocentric velocities, the latter being in one instance as low as 5 km./sec. However, according to Professor C. P. Olivier, who is well acquainted with all the material obtained at the Yale University Observatory,* "later and more numerous, though unpublished, results at Yale frequently gave velocities of the predicted size, and some even hyperbolic."†

(e) The errors in the geocentric velocity of meteors (and consequently in the heliocentric velocity) may be caused by the unknown effect of the resistance of the air and the insufficiently exact estimation of the duration of flight (in his paper Fisher does not reveal any errors in the path-length). Fisher is quite right in stating that the first factor should result in the observed velocity being smaller than the initial one,‡ so that the effect of the second factor must be sufficiently great to produce as a total result velocities which are greater than the true ones. On the other hand, it has long since been found that inexperienced observers (and such as a rule observe fireballs) believe a second to be of a shorter duration than it actually is. Thus, here as well, we are justified in expecting the geocentric velocity of meteors to be underestimated. And indeed Schiaparelli, Niessl, and others have stated that the more reliable the observation of a fireball, the greater the mean velocity which is obtained for it.

It is beyond doubt that taken as a whole the determinations of fireball velocities, and equally of their heights, are liable to contain considerable casual errors. It is most probable that the mean fireball heights obtained are somewhat greater than the true heights (due to the unsymmetrical distribution of casual deviations), and hence somewhat overestimated velocities result. However, we do not believe Fisher's work to be so well grounded as to lead us to reject all the vast material compiled in Niessl-Hoffmeister's catalogue.

3. It has been possible to use for the study of the relation between the heights and the geocentric velocities of fireballs 405 fireballs for

* Ch. P. Olivier, *Report of the American Meteor Society for 1922*, p. 3.

† Ch. P. Olivier, *Meteors*, p. 106.

‡ C. Hoffmeister found that the observed geocentric velocity of the fireball observed on 1916 September 3 was by 31 per cent. smaller than the initial velocity (*Mitteilungen d. Sternwarte zu Sonneberg*, Nr. 14, 1928).

the point of appearance and 407 fireballs for the point of disappearance. In the instances where the catalogue presented two determinations for one and the same fireball we used the more reliable of the two, *i.e.* the one which was based on the greater amount of material or the one that was more recently compiled.

The data of the doubtful cases, marked in the catalogue with signs ?, >, <, etc., were not taken into consideration.

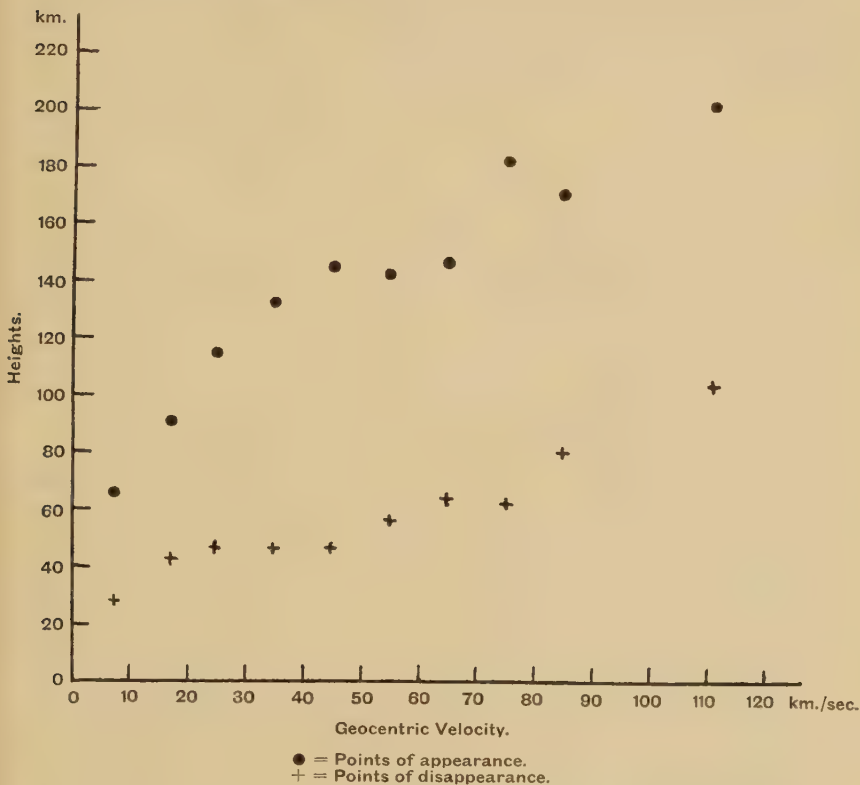


FIG. 2.

All the fireballs were divided into groups according to their geocentric velocities, the latter varying from 0.0–9.9 km./sec., 10.0–19.9 km./sec., etc., within the limits of each group. The mean heights of appearance and of disappearance were then found for each group. The results obtained are shown in Table I. and presented graphically in fig. 2.

As we see, the relation between the geocentric velocity and the height of a fireball is very manifest, both for the points of appearance and the points of disappearance. The greater the relative velocity of a fireball, the higher the point at which it produces incandescence and

that at which it disappears. Thus Niessl's conclusion with regard to the erroneousess of the belief expressed by Schiaparelli, that owing to the resistance of the air the heights at which meteors disappear do not depend on their geocentric velocities, has been fully confirmed. According to the data we obtained, the increase in the heights of the points at which fireballs disappeared as connected with the increase of their geocentric velocities was more rapid than according to Niessl's data obtained from a smaller material. If for convenience we assume a linear relation, we find that the heights of the points at which meteors disappear may be satisfactorily represented by the following formula :

$$H_2 = 0.52v_0 + 30.6,$$

where v_0 is the geocentric velocity, and varies within the limits 10 to 100 km./sec.

TABLE I.

Limits of Geocentric Velocity.	Point of Appearance.			Point of Disappearance.		
	Mean Velocity.	Number of Fireballs.	Height.	Mean Velocity.	Number of Fireballs.	Height.
	km./sec.		km.	km./sec.		km.
0.0-9.9	7.2	4	66.0	7.2	4	28.5
10.0-19.9	16.8	33	91.3	16.9	30	43.5
20.0-29.9	24.6	85	115.1	24.4	88	47.2
30.0-39.9	34.4	100	132.4	34.5	99	46.9
40.0-49.9	44.5	72	145.6	44.5	72	47.6
50.0-59.9	54.6	46	142.4	54.7	49	56.9
60.0-69.9	64.5	29	147.8	64.5	29	65.0
70.0-79.9	74.8	11	183.2	74.8	11	63.8
80.0-89.9	84.4	13	171.7	84.4	13	81.5
90 ≤	110.5	12	203.5	110.5	12	104.7

It seems to us that the data presented in Table I. may further prove to be of use in working out the physical theory of the incandescence of fireballs. The comparison between the results we obtained and the observations of shooting stars shows that the latter usually appear some 20-30 km. lower, and disappear about 20-25 km. higher than fireballs travelling with the same geocentric velocity.

4. It was of interest to find whether or not there was any relation between the heights of fireballs travelling with one and the same geocentric velocity and their heliocentric velocities. For this purpose all the fireballs were divided into three groups: (I.) with heliocentric velocities $v \leq 36$ km./sec. (this group we shall further conditionally term *elliptical*); (II.) with the values of heliocentric velocities $36 < v < 50$ km./sec. (*parabolic* fireballs); and (III.) with values $v \geq 50$ km./sec. (*hyperbolic* fireballs).^{*} For each of these groups the mean heights were determined

^{*} In connection with this I take the liberty of pointing out an error contained in C. Sparrow's paper, "Physical Theory of Meteors," *Ap. J.*, **63**, pp. 90 and 104, 1926. Having computed the theoretical heights of incandescence for meteors of various geocentric velocities, Sparrow states that he does not believe in the

as dependent on their geocentric velocities. The results are shown in Table II. (for the geocentric velocities < 70 km./sec.; all fireballs of greater geocentric velocities are hyperbolic).

TABLE II.

Geocentric Velocity. km./sec.	Height of Appearance.						Height of Disappearance.					
	Hyperbolic.		Parabolic.		Elliptical.		Hyperbolic.		Parabolic.		Elliptical.	
	Number of Fireballs.	H ₁ .	Number of Fireballs.	H ₁ .	Number of Fireballs.	H ₁ .	Number of Fireballs.	H ₂ .	Number of Fireballs.	H ₂ .	Number of Fireballs.	H ₂ .
0.0-9.9	..	..	1	80.0	3	61.3	..	..	1	38.0	3	25.3
0.0-19.9	..	..	26	97.2	7	69.4	..	..	23	42.9	7	45.6
0.0-29.9	26	130.2	47	112.6	12	92.3	24	49.0	52	47.2	12	43.8
0.0-39.9	52	142.3	40	125.0	8	104.4	51	43.9	40	50.4	8	48.9
0.0-49.9	47	156.6	22	125.7	3	119.7	48	41.6	21	56.3	3	84.0
0.0-59.9	37	146.9	5	122.8	4	124.8	39	52.4	7	62.1	3	103.0
0.0-69.9	26	149.9	3	129.0	..	..	26	62.0	3	90.7	..	..

We see that the relation between the height of appearance and the heliocentric velocity is clearly manifest; it is somewhat less distinct for the heights of disappearance. In general, hyperbolic fireballs produce incandescence higher and disappear lower than either parabolic or elliptical fireballs. The latter possess the least heights of appearance, while the heights of their disappearance are somewhat greater than those of the previous two groups.

The geocentric velocities being equal, the heights of fireballs may depend on the angle of inclination (on the zenith distance of a radiant), on mass and chemical composition. The first factor does not explain the dependence which we have established, as the mean values of inclination for different groups of fireballs vary within the inconsiderable limits of from 55° to 65° . Consequently the fireballs included in our three groups differ from each other either in mass or in chemical composition.

We consider the following explanation to be the most probable: *fireballs with greater heliocentric velocities have a larger mass; the greater the mass of a meteor the higher its point of appearance and the lower the point of its disappearance.* Unfortunately the existing physical theories give no satisfactory answer as to the dependence of the heights of the

reality of the hyperbolic velocities of meteors, on the ground of the assumption that if these were true the heights of their appearance should have been greater than those of the Leonids (*i.e.* greater than 150 km.), which is, however, observed but rarely. In making this statement Sparrow seems to neglect the consideration that he is using the *geocentric* velocity, which is rather great for the Leonids (76 km./sec.). Meteors with hyperbolic *heliocentric* velocities possess a lesser geocentric velocity than that of the Leonids. Thus out of the total number of 224 fireballs with a heliocentric velocity ≥ 50 km./sec., *i.e.* of obviously hyperbolic fireballs, only 28 (12.5 per cent.) possess a geocentric velocity which is greater than that of the Leonids. Hence the naturally small percentage of meteors with heights of appearance greater than 150-160 km.

points of incandescence of meteors on their masses (their geocentric velocities being equal).^{*} The following facts seem to be in favour of our assumption :—

(a) It is beyond doubt that ordinary shooting stars are possessed of a smaller mass than fireballs and meteorites. Nevertheless, according to Niessl,[†] the mean height of the appearance of shooting stars is 108.5 km., while that of fireballs is 138.6 km.; the heights of disappearance are respectively 86.3 and 49.7 km. Thus if we assume that the mean geocentric velocity is the same, our assumption is confirmed.

Similar results are obtained by comparing the heights of the appearance of fireballs with those of well-known shower radiants :

	Mean Geocentric Velocity. km./sec.	Shower Radiants. km.	Fireballs. km.
(a)	76	154	175
(b)	54	115	144
(c)	33	97	120

(a, the Leonids ; b, the Perseids ; c, the Geminids.)

The values of the fireball heights have been obtained here by means of a graphical interpolation in fig. 1.

TABLE III.

Geocentric Velocity. km./sec.	Heliocentric Velocity.	Brightness.				
		π	ρ	Part ζ	ζ	$\odot$
20.0-29.9	Hyperbolic	24	32	20	24	0
		6	8	5	6	0
	Parabolic	49	21	19	6	4
		23	10	9	3	2
	Elliptical	42	33	0	25	0
		5	4	0	3	0
30.0-39.9	Hyperbolic	17½	17½	27½	32½	5
		7	7	11	13	2
	Parabolic	25	30½	25	19½	0
		9	11	9	7	0
	Elliptical	22	44	22	12	0
		2	4	2	1	0
40.0-49.9	Hyperbolic	29	17	21	19	14
		12	7	9	8	6
	Parabolic	55	15	15	10	5
		11	3	3	2	1
	Elliptical	50	25	25	0	0
		2	1	1	0	0

(b) Let us also consider the distribution of fireballs of various geocentric velocities according to their brightness. We shall limit

^{*} Cf. F. A. Lindemann, "Note on the Physical Theory of Meteors," *Ap. J.*, 65, 120, 1927.

[†] *Op. cit.*, p. 453.

ourselves to the consideration of fireballs whose geocentric velocities vary from 20 to 50 km./sec., as within these limits all the three groups are represented with sufficient completeness (Table III.). (The figures in italics show in percentages the distribution of fireballs according to their brightness within each of the three groups; the lower figures show the corresponding number of fireballs.)

We see that, in general, fireballs with a greater heliocentric velocity have, on the average, a greater brightness. Indeed, for the 252 fireballs considered, we have :

Heliocentric Velocity.	Number of Fireballs.	Per cent. of Fireballs.			
		≤ 9	$10 \leq$	Detonating Fireballs.	Meteorites.
Hyperbolic	114	44	<i>31½</i>	23	4
Parabolic	111	65	15	11	0
Elliptical	27	72	12	6	0

The preponderance of greater brightnesses, as well as of detonating fireballs and of meteorites in the group of hyperbolic fireballs, is quite obvious; hence it follows (if we take into consideration the equal geocentric velocities and the inconsiderable difference in the mean distances separating the observer from the various groups of fireballs) that our assumption as to hyperbolic fireballs being possessed of a greater mass is correct.

Summary.

1. Fisher's conclusion (*Harv. Circ.*, 331, 1928) as to the erroneous-ness of the hyperbolic velocities of fireballs appears to be insufficiently grounded.

2. Niessl's conclusion with regard to the increase in the heights of the appearance and disappearance of fireballs as dependent on the increase of their geocentric velocity (see Table I. and fig. 2) is fully confirmed.

3. The geocentric velocities being equal, the heights of the appearance and disappearance of fireballs depend on their heliocentric velocities. The greater the heliocentric velocity the higher the point at which the fireballs appear and the lower that at which they disappear.

This may be accounted for by the hypothesis that :

(a) Fireballs having greater heliocentric velocities are, on the average, of greater mass; (b) the height of the appearance of meteors depends on their mass. Meteors possessed of a greater mass (other conditions being equal) appear at a greater height.

Mirovédénjé-Society, Leningrad :
1929 September 1.

Occultations of Stars by the Moon, observed at Tomsk University Observatory during the Year 1929. By N. Goryatscheff.

(Communicated by the Secretaries.)

$(\phi = \text{N. } 56^{\circ} 28' 07''; l = 5^{\text{h}} 39^{\text{m}} 47^{\text{s}}.4 \text{ East of Greenwich.})$

Date (U.T.).	Local Sidereal Time.			Star.	Mag.	Phase.
1929.	h	m	s			
II. 17, 554634	4	46	34.8	51 Tauri	5.6	D
II. 17, 580197	5	23	29.5	56 Tauri	5.2	D
II. 18, 751532	9	34	49.9	118 Tauri	5.4	D
III. 15, 602088	7	37	36.5	B.D.+16°.392	8.2	D
III. 15, 617575	7	59	58.2	B.D.+16°.395	8.8	D
III. 15, 625051	8	10	45.9	B.D.+16°.398	8.2	D
III. 18, 751873	11	25	42.9	Mars (2 cont.)	..	D
III. 18, 784530	12	12	52.2	Mars (4 cont.)	..	R
III. 19, 596740	7	45	39.4	49 Geminorum	6.9	D
IV. 15, 615899	9	59	46.1	39 Geminorum	6.2	D
IV. 15, 654758	10	55	52.7	B.D.+26°.1424	8.8	D
IV. 15, 658684	11	01	32.9	B.D.+26°.1426	8.6	D
IV. 17, 636576	10	37	30.6	B.D.+22°.2039	8.2	D
V. 13, 698967	13	50	06.4	B.D.+25°.1768	8.5	D
V. 13, 724580	14	27	05.3	B.D.+25°.1773	8.3	D
V. 13, 734696	14	41	41.8	B.D.+25°.1778	8.0	D
VII. 29, 832091	22	05	54.8	B.D.+13°.410	8.3	R
VII. 29, 841454	22	19	25.9	B.D.+13°.411	7.6	R
VII. 29, 846076	22	26	06.4	B.D.+13°.413	8.4	R
VII. 30, 852066	22	38	41.9	B.D.+18°.492	8.3	R
VII. 30, 866203	22	59	06.7	B.D.+18°.494	8.1	R
VII. 31, 771445	20	46	13.7	67 Tauri	5.4	R
VII. 31, 776432	20	53	25.8	κ Tauri	4.1	R
VII. 31, 879539	23	22	18.6	B.D.+22°.707	7.9	R
VIII. 1, 795849	21	25	24.6	118 Tauri	5.4	R
VIII. 24, 734650	21	27	43.2	μ Piscium	5.0	R
VIII. 29, 827144	0	00	59.4	B.D.+26°.1150	8.5	R
VIII. 29, 841414	0	21	35.7	B.D.+26°.1156	8.2	R
VIII. 29, 862187	0	51	35.4	B.D.+26°.1169	8.0	R
VIII. 29, 883592	1	22	29.8	B.D.+26°.1181	8.9	R
VIII. 29, 901547	1	48	25.4	B.D.+26°.1185	8.4	R
IX. 23, 987478	5	31	04.0	A Tauri	4.5	R
IX. 26, 730656	23	32	03.5	B.D.+26°.1404	8.6	R
X. 16, 693180	23	56	47.7	25 B Ceti	6.8	D

Apparent Places of Occulted Stars of Magnitude > 6.5.

Star.	α .			δ .	Authority.
	h	m	s		
B.D. +16° 392	3	04	05.94	+16° 39' 13".2	Bord. ph. Berl. A.
B.D. +16° 395	3	05	37.79	+16 38 06.5	
B.D. +16° 398	3	06	17.63	+16 32 42.2	
49 Geminorum	7	08	27.82	+25 52 12.5	Gr ₁₉₁₀ 3643; WZC. 495.
B.D. +26° 1424	6	56	50.14	+26 17 16.4	Gr ₁₉₁₀ 3519.
B.D. +26° 1426	6	57	01.81	+26 20 17.0	Gr ₁₉₁₀ 3523.
B.D. +22° 2039	8	58	40.97	+21 48 02.3	Abbadia 5360.
B.D. +25° 1768	7	45	12.64	+25 16 22.8	Gr ₁₉₁₀ 3980.
B.D. +25° 1773	7	46	39.67	+25 17 34.6	Gr ₁₉₁₀ 3999.
B.D. +25° 1778	7	47	30.38	+25 00 21.3	Gr ₁₉₁₀ 4005.
B.D. +13° 410	2	31	50.64	+13 30 35.2	Bord. ph. Lpz. I.
B.D. +13° 411	2	32	09.62	+13 26 55.5	
B.D. +13° 413	2	32	28.08	+13 21 46.4	
B.D. +18° 492	3	26	52.40	+18 33 40.5	Abbadia 1543.
B.D. +18° 494	3	27	40.96	+18 33 40.6	Abbadia 1554.
B.D. +22° 707	4	27	21.09	+22 41 44.4	Abbadia 2090.
B.D. +26° 1150	6	11	28.40	+26 26 32.7	Gr ₁₉₁₀ 2968.
B.D. +26° 1156	6	12	26.77	+26 31 12.8	Gr ₁₉₁₀ 2984.
B.D. +26° 1169	6	13	42.08	+26 27 50.3	Gr ₁₉₁₀ 2994.
B.D. +26° 1181	6	15	21.62	+26 42 40.4	Gr ₁₉₁₀ 3011.
B.D. +26° 1185	6	16	06.49	+26 36 15.3	Gr ₁₉₁₀ 3025.
B.D. +26° 1404	6	53	30.56	+26 39 53.6	Gr ₁₉₁₀ 3478.
25 B. Ceti	0	07	34.36	- 3 42 39.4	Abbadia 27; WZC. 7.

The proper motions are computed from Schorr, *Eigenbewegungslexikon*.

The places of Mars are computed from the *Nautical Almanac*.

The time error of the Moon is respectively :

+ 15^s.7; + 11^s.6; + 10^s.6; + 10^s.2; + 11^s.4; + 12^s.9; + 17^s.3;
 + 12^s.5; + 13^s.5; + 16^s.9; + 13^s.7; + 13^s.6; + 13^s.6; + 11^s.5;
 + 10^s.8; + 11^s.5; + 6^s.5; + 13^s.4; + 7^s.1; + 6^s.6; + 9^s.7; + 12^s.0;
 + 11^s.2; + 10^s.2; + 11^s.3; + 15^s.7; + 14^s.9; + 13^s.0; + 12^s.3;
 + 9^s.1; + 12^s.0; + 14^s.6; + 10^s.6; + 12^s.0.

Observer.—N. Goryatscheff.

Instruments.—Zeiss telescope, 90-mm. aperture; magnifying power 90. Observations made with eye and ear.

The time-corrections of the chronometer were found from the Bordeaux, Rugby, and Nauen wireless signals.

Tomsk, Siberia, U.S.S.R.,
 University Astronomical Observatory :
 1930 January 21.

Erratum.

Page 473. R.A. of star B.D. + 6° 1309 (1923),—for 6^h 30^m 42^s.93 read 6^h 30^m 42^s.01.

